Running Head: RELATIONS

Week 5 Application: Relations

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Week 5 Application: Relations

Graded questions: 5.1-12, 5.1-21, 5.2-1, 5.3-20, and 5.4-3.

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| 5.1-12  Find the inverse.  Let U = ***R***, where R is defined by x R y iff x < y.  Inverse.  Let U = ***R***, where R-1 is defined by y R-1 x iff x < y.  That is y > x.  Therefore, the inverse of the “Is Less Than” relation <, is the “Is Greater Than” relation >. |
| 5.1-21  Draw: Given the set {0,1,2}, {0,1,2} R P({0,1,2}) iff {0,1,2} ∈ P(x).   |  |  | | --- | --- | | Element | P(x) | | 0 | {0,1,2} | |  | {0,1} | |  | {0} | | 1 | {1} | |  | {1,2} | |  | {2} | | 2 | {0,2} | |  | θ | |
| 5.2-1  Let X ⊆ Z+, prove that R as defined on X by a R b ↔ a | b is a partial order relation.  Reflexive: Let x ∈ X, since x = 1x then x | x that is x R x.  Antisymmetric: Let x, y ∈ X, suppose x | y and y | x, since x, y ≠ 0, x = ±y and y=±x, then x = y.  Transitive: Let x, y, z ∈ X, suppose x | y and y | z, that is x | z.  Therefore, R as defined on X by a R b ↔ a | b is a partial order relation. |

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| 5.3-20  What is the domain and range of  F:{0,1,…,4}→{0,1,…,25} defined by n ↦ n!.  Domain: {0,1,…,4}  Range: {1, 2, 6, 24}   |  |  | | --- | --- | | f(x)=n! | | | X | Y | | 0 | 0 | | 1 | 1 | | 2 | 2 | | 3 | 6 | | 4 | 24 | |  | 25 | |
| 5.4-3  “Show that f(x)=x3+8 is one-to-one” (Ferland, 2009, p. 270).  Suppose that ∀ n1, n2 ∈ X.  Such that f(n1)=f(n2)  That is n13+8= n23+8  n13= n23  n1= n2  Therefore, f(x)=x3+8 is injective. |

Section 5.1 Odd Questions 1-17 and 21-29 (Note, question 21 is located at the top of this paper).

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| 5.1-1  “Let X be the set of primes, let Y = Z, and define the relation R from X to Y by a R b if and only if a|b” (Ferland, 2009, p. 230).   |  |  |  | | --- | --- | --- | | A. True or False: 5 R 1  Suppose 5 R 1.  That is, 5|1.  However, 5|1 is false.  Since a|b must be true.  5 R 1 is false. | B. True or False: 3 R 6  Suppose 3 R 6.  That is, 3|6.  Observe 6/3=2,  So3|6 is true.  Since a|b must be true.  3 R 6 is true. | C. True or False: 2 R 7  Suppose 2 R 7.  That is, 2|7  However, 2|7 is false.  Since a|b must be true.  2 R 7 is false. | |
| 5.1-3  “Given the universal set U = ***R***, let X = P(U), and define the relation R on X by A R B if and only if A⊆B” (Ferland, 2009, p. 231).   |  |  |  | | --- | --- | --- | | A. True or False: θ R Z.  Suppose θ R Z.  That is, θ ⊆ Z.  Such that, θ ∈ Z.  Since θ ∈ Z, and Z ∈ R.  θ R Z is True. | B. True or False: 0 R θ.  Suppose 0 R θ.  That is, 0 ⊆ θ.  Such that, 0 ∈ θ.  Since 0 ∉ θ.  0 R θ is False. | C. True or False: {1,2} R ***R+.***  Suppose {1,2} R ***R+.***  That is, {1,2} ⊆ ***R+.***  Such that, {1,2} ∈ ***R+.***  Since {1,2} ∈ ***R+,***and ***R+*** ∈ ***R.***  {1,2} R ***R+*** is True. | |
| 5.1-5  True or False: “The ‘is a superset of’ relation ⊇ is the inverse of ⊆” (Ferland, 2009, p. 231)?  Let B be an arbitrary set of U, and a be an element of U.  Observe that a ⊆ B, such that a ∈ B, or B ∍ a.  Also, B ⊇ a, such that B ∍ a, or a ∈ B.  That is a ⊆ B ≡ B ⊇ a.  Therefore, ⊇ is the inverse of ⊆ is a True statement. |
| 5.1-7  Where U = R2, is ⊥ R-1 || True or False?  False. |
| 5.1-9  Using the country set as shown on page 231 of Discrete Mathematics.  Where U = South America, R is defined by A R B iff A shares a border with B.   |  |  | | --- | --- | | A. True or False: Chile R Paraguay.  Chile R Argentina, Bolivia, Peru.  Paraguay R Argentina, Bolivia, Brazil.  Note that Chile does not share a border with Paraguay.  Since A must share a border with B.  Chile R Paraguay is False. | B. List all x where Venezuela R x.  Venezuela R Guyana, Brazil, Colombia.  That is: Guyana R Venezuela; Brazil R Venezuela; Colombia R Venezuela. | |
| 5.1-11  Find the inverse.  The “is father of” relation.  Let U be arbitrary, R is defined by A R B iff A is a parent of B.  The inverse.  Let U be arbitrary, R-1 is defined by B R A iff A is a parent of B.  That is B is a child of A.  The “is child of” relation. |

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| 5.1-13  Find the inverse.  The “is a subset of” relation ⊆.  Let U = ***R***, let X = P(U), R is defined on X by A R B iff A⊆B.  The inverse.  Let U = R, let X = P(U), R-1 is defined on X by B R A iff A ⊆ B.  That is B ⊇ A.  The “is a superset of” relation ⊇. |
| 5.1-15  Find the inverse.  The “is parallel to” relation ||.  Let U = R2, R is defined by A R B iff A is parallel to B.  The Inverse.  Let U = R2, R is defined by B R-1 A iff A is parallel to B.  That is, B is perpendicular to A ⊥.  The “is perpendicular to” relation ⊥. |
| 5.1-17  Find the inverse.  The relation R on ***R*** defined by x R y iff x2+y2=1.  The inverse.  The relation R on R defined by y R-1 x iff x2+y2=1.  Since R is symmetric (e.g. y2+x2= x2+y2, and both must equal 1).  R |

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| 5.1-23  Draw: Given the set {Skate Rats, NBA Dunkfest, Rx Tracker} as Projects, and the set {Medi Comp, GameCo} as Clients, Projects R Clients iff Projects are being done for specified Client.   |  |  | | --- | --- | | Projects | Client | | Skate Rats | Medi Comp | | NBA Dunkfest |  | | Rx Tracker | GameCo. | |
| 5.1-25  Draw: The relation < on {2,4,6,8}. |

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| 5.1-27  Draw: The ⊆ relation on {{1},{2},{1,2},{2,3},{1,2,3}}. |
| 5.1-29 |

Section 5.2 Odd Questions 1-5

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| 5.2-3  Prove: R-1 is a partial order relation on X, when R is a partial order relation on X.  Reflexive: Let a ∈ X. Suppose a R a, such that a R-1 a.  Antisymmetric: Let a, b ∈ X. Suppose a R b and b R a. That is b R-1 a and a R-1 b. Such that, a = b.  Transitive: Let a, b, c ∈ X. Suppose a R b and b R c, such that c R-1 b and b R-1 a.  Specifically a R c and c R-1 a.  Therefore, R-1 is a partial order relation on X, when R is a partial order relation on X. |
| 5.2-5  Question 53: Let U = ***R***, let X = P(U), R is a relation on X by A R B ↔ x ∈ A.  Reflexive: Let x ∈ X. Suppose x ∈ A and x ∈ B that is x∈A R x ∈ B.  Antisymmetric: Let x, y ∈ X. Suppose x ∈ A and A R B, such that x ∈ B.  That is B R A, since x = x, A = B.  Transitive: Let x ∈ X. Suppose x ∈ A and A R B, such that x ∈ B, also B R C such that x ∈ C.  Specifically x ∈ A R C.  Therefore, R is a partial order relation on X by A R B ↔ x ∈ A. |

Section 5.3 Odd Questions 1-9 and 17-25.

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| 5.3-1  “Is codomain synonymous with range” (Ferland, 2009, p. 257)?  Given by definition 5.13: “The codomain, or target, of f is the set Y” (Ferland, 2009, p. 251). Also, “the range, or image, of f is the set range(f) = {y:y ∈ Y and f (x)= y for some x ∈ X}” (Ferland, 2009, p. 251).  Therefore, no, the codomain is not synonymous or equivalent to the range of a function. |
| 5.3-3  Given f from Z to Z+ by f(n) = 2n-1, is this a defined function?  Let n = 0.  Observe: f(0)=20-1=1/2.  ½ ∉ Z.  Therefore, f(n) = 2n-1, is not a well-defined function. |
| 5.3-5  Given f from [0, ∞) to R by x ↦ ± , is this a defined function?  Let x = 100, and x ∈ X. Also, Let f(x)=y where y ∈ Y.  Observe that f(100)== -10 or +10.  Note that X can map to Y only once.  x = -10 and +10.  Therefore, x ↦ ± , is not a function. |

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| 5.3-7  Define f from Z → Z by:   |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | A. f(n)= n/2  Let n = 1.  Observe that f(n)=1/2.  That is, f(1) ∉ Z.  Therefore, f(n)= n/2 is not a well-defined function. | B. f(n)=2n  Suppose not (f(n) is not a function).  Let x ∈ X, such that, f(x)=2x ∈ Y.  Induction: (goal f(x+1) ∉ Z)  Observe: f(x+1)=2(x+1)  =2x+2  =x + x + 2  > 2x ∈ Y  Y ∈ Z.  This is a contradiction.   |  |  | | --- | --- | | f(x)=2x | | | A | B | | 0 | 0 | | 1 | 2 | | n | 2n |   Therefore, f(n)=2n is a well-defined function. | |
| 5.3-9  Given f from Z to Q by n ↦ is this a defined function?  Suppose not.    Observe that  Also, f(x)=y returns a single value for each element of x.  Therefore, n ↦ is a well-defined function. |

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| 5.3-17  What is the Domain and Range of  f: {-3, -2,…, 3} → {-10, -9,…, 10} defined by n ↦ n2.  Domain: {-3, -2,…, 3}  Range: {0, 1, 4, 9} |
| 5.3-19  What is the domain and range of  F:{0, 1, …, 4} → {0, 1,…,25} defined by n ↦ 2n.  Domain: {0, 1, …, 4}  Range: {1, 2, 8, 16} |
| 5.3-21  What is the domain and range of  f: R → R defined by x ↦ x2-1.  Domain: R  Range: [1, ∞) |
| 5.3-23  What is the domain and range of  f(x) =  Domain: [1, ∞)  Range: [0,∞) |
| 5.3-25  What is the domain and range of  f(x)=  Domain: R\{-1}  Range: R\{0} |

Section 5.4 Odd Questions 3-11 (Note that question 3 is located at the top of this paper).

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| 5.4-5  “Show that the function f: R+ → R+ defined by x ↦ x2+x is onto” (Ferland, 2009, p. 270).  Let y ∈ Y.  Such that f(x)=y= x2+x  Observe that ∀ y ∈ R+, = x.  That is, f(y)=x.  So that, Y ⊆ range(f(x)).  Therefore, x ↦ x2+x is surjective. |
| 5.4-7  Prove: f: R → R defined by f(x) < f(y) is injective.  Let n1, n2 ∈ X.  Suppose n1 ≠ n2.  So n1 < n2 or n2 < n1.  Such that f(n1) < f(n2) or f(n2) < f(n1).  Specifically n1 ≠ n2.  Therefore, f(x) < f(y) is not injective. |

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| 5.4-9  Given f: Z6 → Z6 by [n] ↦ [2n]   |  |  |  | | --- | --- | --- | | A. Is f well defined?  Let x1, x2 ∈ X.  So, [x1] = [x2]  Since Z6, 6|(x1-x2)  That is 6|(2x1-2x2)  Thus, [2x1] = [2x2].  Therefore, [n] ↦ [2n] is well defined (Ferland, 2009, p. A30). | B. Is f injective?  Observe that “f([0])=f([3])” (Ferland, 2009, p. A30). | C. Does cancellation work with f?  “gcd(2,6) ≠ 1” (Ferland, 2009, p. A30)  This question absolutely stumped me. I am not sure what the function as given means. | |
| 5.4-11  Given h(n)=n mod 625   |  |  | | --- | --- | | A. Hash: 0 16000 81160 7  h(01600081160)= 535  535=1600081160 (mod 625)  625|(535 – 1600081160)  625|-1600080625  Therefore, the hash is 535. | B. Give two ISBN numbers which hash to 321.  0 00000 00321 6  h(321)=321 (mod 625)  0 00000 07821 4  h(7821)=321 (mod 625) | |

Reference

Ferland, K. (2009). *Discrete Mathematics.* Boston: Houghton Mifflin Company.