Running Head: RELATIONS

Week 5 Application: Relations

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Week 5 Application: Relations

Graded questions: 5.1-12, 5.1-21, 5.2-1, 5.3-20, and 5.4-3.

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| 5.1-12Find the inverse.Let U = ***R***, where R is defined by x R y iff x < y. Inverse. Let U = ***R***, where R-1 is defined by y R-1 x iff x < y. That is y > x.Therefore, the inverse of the “Is Less Than” relation <, is the “Is Greater Than” relation >. |
| 5.1-21Draw: Given the set {0,1,2}, {0,1,2} R P({0,1,2}) iff {0,1,2} ∈ P(x).

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| Element  | P(x) |
| 0 | {0,1,2} |
|  | {0,1} |
|  | {0} |
| 1 | {1} |
|  | {1,2} |
|  | {2} |
| 2 | {0,2} |
|  | θ |

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| 5.2-1Let X ⊆ Z+, prove that R as defined on X by a R b ↔ a | b is a partial order relation. Reflexive: Let x ∈ X, since x = 1x then x | x that is x R x. Antisymmetric: Let x, y ∈ X, suppose x | y and y | x, since x, y ≠ 0, x = ±y and y=±x, then x = y. Transitive: Let x, y, z ∈ X, suppose x | y and y | z, that is x | z.Therefore, R as defined on X by a R b ↔ a | b is a partial order relation. |

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| 5.3-20What is the domain and range of F:{0,1,…,4}→{0,1,…,25} defined by n ↦ n!. Domain: {0,1,…,4} Range: {1, 2, 6, 24}

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| f(x)=n! |
| X | Y |
| 0 | 0 |
| 1 | 1 |
| 2 | 2 |
| 3 | 6 |
| 4 | 24 |
|  | 25 |

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| 5.4-3“Show that f(x)=x3+8 is one-to-one” (Ferland, 2009, p. 270). Suppose that ∀ n1, n2 ∈ X. Such that f(n1)=f(n2) That is n13+8= n23+8 n13= n23 n1= n2Therefore, f(x)=x3+8 is injective. |

Section 5.1 Odd Questions 1-17 and 21-29 (Note, question 21 is located at the top of this paper).

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| 5.1-1“Let X be the set of primes, let Y = Z, and define the relation R from X to Y by a R b if and only if a|b” (Ferland, 2009, p. 230).

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| A. True or False: 5 R 1Suppose 5 R 1. That is, 5|1. However, 5|1 is false. Since a|b must be true.5 R 1 is false.  | B. True or False: 3 R 6Suppose 3 R 6. That is, 3|6. Observe 6/3=2,  So3|6 is true. Since a|b must be true.3 R 6 is true. | C. True or False: 2 R 7Suppose 2 R 7. That is, 2|7 However, 2|7 is false. Since a|b must be true.2 R 7 is false. |

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| 5.1-3“Given the universal set U = ***R***, let X = P(U), and define the relation R on X by A R B if and only if A⊆B” (Ferland, 2009, p. 231).

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| A. True or False: θ R Z.Suppose θ R Z. That is, θ ⊆ Z. Such that, θ ∈ Z. Since θ ∈ Z, and Z ∈ R.θ R Z is True. | B. True or False: 0 R θ.Suppose 0 R θ. That is, 0 ⊆ θ. Such that, 0 ∈ θ. Since 0 ∉ θ.0 R θ is False. | C. True or False: {1,2} R ***R+.***Suppose {1,2} R ***R+.***That is, {1,2} ⊆ ***R+.*** Such that, {1,2} ∈ ***R+.***Since {1,2} ∈ ***R+,***and ***R+*** ∈ ***R.***{1,2} R ***R+*** is True. |

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| 5.1-5True or False: “The ‘is a superset of’ relation ⊇ is the inverse of ⊆” (Ferland, 2009, p. 231)?Let B be an arbitrary set of U, and a be an element of U. Observe that a ⊆ B, such that a ∈ B, or B ∍ a. Also, B ⊇ a, such that B ∍ a, or a ∈ B. That is a ⊆ B ≡ B ⊇ a.Therefore, ⊇ is the inverse of ⊆ is a True statement. |
| 5.1-7Where U = R2, is ⊥ R-1 || True or False?False. |
| 5.1-9Using the country set as shown on page 231 of Discrete Mathematics.Where U = South America, R is defined by A R B iff A shares a border with B.

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| A. True or False: Chile R Paraguay.Chile R Argentina, Bolivia, Peru. Paraguay R Argentina, Bolivia, Brazil.Note that Chile does not share a border with Paraguay. Since A must share a border with B.Chile R Paraguay is False. | B. List all x where Venezuela R x.Venezuela R Guyana, Brazil, Colombia. That is: Guyana R Venezuela; Brazil R Venezuela; Colombia R Venezuela. |

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| 5.1-11Find the inverse.The “is father of” relation. Let U be arbitrary, R is defined by A R B iff A is a parent of B. The inverse. Let U be arbitrary, R-1 is defined by B R A iff A is a parent of B. That is B is a child of A.The “is child of” relation. |

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| 5.1-13Find the inverse.The “is a subset of” relation ⊆. Let U = ***R***, let X = P(U), R is defined on X by A R B iff A⊆B. The inverse. Let U = R, let X = P(U), R-1 is defined on X by B R A iff A ⊆ B. That is B ⊇ A.The “is a superset of” relation ⊇. |
| 5.1-15Find the inverse.The “is parallel to” relation ||. Let U = R2, R is defined by A R B iff A is parallel to B. The Inverse. Let U = R2, R is defined by B R-1 A iff A is parallel to B. That is, B is perpendicular to A ⊥.The “is perpendicular to” relation ⊥. |
| 5.1-17Find the inverse.The relation R on ***R*** defined by x R y iff x2+y2=1. The inverse. The relation R on R defined by y R-1 x iff x2+y2=1. Since R is symmetric (e.g. y2+x2= x2+y2, and both must equal 1).R |

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| 5.1-23Draw: Given the set {Skate Rats, NBA Dunkfest, Rx Tracker} as Projects, and the set {Medi Comp, GameCo} as Clients, Projects R Clients iff Projects are being done for specified Client.

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| Projects | Client |
| Skate Rats | Medi Comp |
| NBA Dunkfest |  |
| Rx Tracker | GameCo. |

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| 5.1-25Draw: The relation < on {2,4,6,8}. |

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| 5.1-27Draw: The ⊆ relation on {{1},{2},{1,2},{2,3},{1,2,3}}. |
| 5.1-29 |

Section 5.2 Odd Questions 1-5

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| 5.2-3Prove: R-1 is a partial order relation on X, when R is a partial order relation on X. Reflexive: Let a ∈ X. Suppose a R a, such that a R-1 a. Antisymmetric: Let a, b ∈ X. Suppose a R b and b R a. That is b R-1 a and a R-1 b. Such that, a = b. Transitive: Let a, b, c ∈ X. Suppose a R b and b R c, such that c R-1 b and b R-1 a.  Specifically a R c and c R-1 a.Therefore, R-1 is a partial order relation on X, when R is a partial order relation on X. |
| 5.2-5Question 53: Let U = ***R***, let X = P(U), R is a relation on X by A R B ↔ x ∈ A. Reflexive: Let x ∈ X. Suppose x ∈ A and x ∈ B that is x∈A R x ∈ B. Antisymmetric: Let x, y ∈ X. Suppose x ∈ A and A R B, such that x ∈ B.  That is B R A, since x = x, A = B. Transitive: Let x ∈ X. Suppose x ∈ A and A R B, such that x ∈ B, also B R C such that x ∈ C.  Specifically x ∈ A R C.Therefore, R is a partial order relation on X by A R B ↔ x ∈ A. |

Section 5.3 Odd Questions 1-9 and 17-25.

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| 5.3-1“Is codomain synonymous with range” (Ferland, 2009, p. 257)?Given by definition 5.13: “The codomain, or target, of f is the set Y” (Ferland, 2009, p. 251). Also, “the range, or image, of f is the set range(f) = {y:y ∈ Y and f (x)= y for some x ∈ X}” (Ferland, 2009, p. 251).Therefore, no, the codomain is not synonymous or equivalent to the range of a function. |
| 5.3-3Given f from Z to Z+ by f(n) = 2n-1, is this a defined function? Let n = 0. Observe: f(0)=20-1=1/2. ½ ∉ Z.Therefore, f(n) = 2n-1, is not a well-defined function. |
| 5.3-5Given f from [0, ∞) to R by x ↦ ± , is this a defined function? Let x = 100, and x ∈ X. Also, Let f(x)=y where y ∈ Y. Observe that f(100)== -10 or +10. Note that X can map to Y only once. x = -10 and +10.Therefore, x ↦ ± , is not a function. |

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| 5.3-7Define f from Z → Z by:

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| A. f(n)= n/2Let n = 1. Observe that f(n)=1/2. That is, f(1) ∉ Z.Therefore, f(n)= n/2 is not a well-defined function. | B. f(n)=2nSuppose not (f(n) is not a function). Let x ∈ X, such that, f(x)=2x ∈ Y. Induction: (goal f(x+1) ∉ Z) Observe: f(x+1)=2(x+1) =2x+2 =x + x + 2 > 2x ∈ Y Y ∈ Z. This is a contradiction.

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| f(x)=2x |
| A | B |
| 0 | 0 |
| 1 | 2 |
| n | 2n |

Therefore, f(n)=2n is a well-defined function.  |

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| 5.3-9Given f from Z to Q by n ↦ is this a defined function?Suppose not.  Observe that  Also, f(x)=y returns a single value for each element of x.Therefore, n ↦ is a well-defined function. |

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| 5.3-17What is the Domain and Range of f: {-3, -2,…, 3} → {-10, -9,…, 10} defined by n ↦ n2. Domain: {-3, -2,…, 3} Range: {0, 1, 4, 9} |
| 5.3-19What is the domain and range of F:{0, 1, …, 4} → {0, 1,…,25} defined by n ↦ 2n. Domain: {0, 1, …, 4} Range: {1, 2, 8, 16} |
| 5.3-21What is the domain and range of f: R → R defined by x ↦ x2-1. Domain: R Range: [1, ∞) |
| 5.3-23What is the domain and range of f(x) = Domain: [1, ∞) Range: [0,∞) |
| 5.3-25What is the domain and range of f(x)= Domain: R\{-1} Range: R\{0} |

Section 5.4 Odd Questions 3-11 (Note that question 3 is located at the top of this paper).

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| 5.4-5“Show that the function f: R+ → R+ defined by x ↦ x2+x is onto” (Ferland, 2009, p. 270). Let y ∈ Y. Such that f(x)=y= x2+x Observe that ∀ y ∈ R+, = x. That is, f(y)=x. So that, Y ⊆ range(f(x)).Therefore, x ↦ x2+x is surjective. |
| 5.4-7Prove: f: R → R defined by f(x) < f(y) is injective. Let n1, n2 ∈ X. Suppose n1 ≠ n2. So n1 < n2 or n2 < n1. Such that f(n1) < f(n2) or f(n2) < f(n1). Specifically n1 ≠ n2.Therefore, f(x) < f(y) is not injective. |

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| 5.4-9Given f: Z6 → Z6 by [n] ↦ [2n]

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| A. Is f well defined?Let x1, x2 ∈ X. So, [x1] = [x2] Since Z6, 6|(x1-x2) That is 6|(2x1-2x2) Thus, [2x1] = [2x2].Therefore, [n] ↦ [2n] is well defined (Ferland, 2009, p. A30). | B. Is f injective?Observe that “f([0])=f([3])” (Ferland, 2009, p. A30). | C. Does cancellation work with f?“gcd(2,6) ≠ 1” (Ferland, 2009, p. A30)This question absolutely stumped me. I am not sure what the function as given means. |

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| 5.4-11Given h(n)=n mod 625

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| A. Hash: 0 16000 81160 7 h(01600081160)= 535 535=1600081160 (mod 625) 625|(535 – 1600081160) 625|-1600080625Therefore, the hash is 535. | B. Give two ISBN numbers which hash to 321.0 00000 00321 6 h(321)=321 (mod 625)0 00000 07821 4 h(7821)=321 (mod 625) |

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Reference

Ferland, K. (2009). *Discrete Mathematics.* Boston: Houghton Mifflin Company.