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Week 4 Application: Indexed by Integers

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Graded problems: 4.1-4, 4.1-25, 4.2-8, 4.3-9, and 4.4-5.

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| 4.1-4Show:  |
| 4.1-25What is the closed sequence for: 1, -3, 5, -7, 9…? ∀ n ≥ 0, Sn = -1n(1+2n) S0=-10(1+2(0))= 1 S1=-11(1+2(1))= -3 S2=-12(1+2(2))= 5 S3=-13(1+2(3))= -7 S4=-14(1+2(4))= 9 |
| 4.2-8“The sum of the odd integers from 5 to 1001” (Ferland, 2009, p. 182).Solve and show the sigma notation for: 1+3+5+7+…+1001. |

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| 4.3-9Prove: ∀ n ≥ 4, n! > 2n.Let n=5. Observe: (5!>25)=120>32.Induction Suppose k ≥ 4, k! > 2k. (Goal: (k+1)! > 2k+1). Observe that: (k+1)! = (k+1)k! > 2k(k+1) > 2k(2) = 2k+1. That is, (k+1)! > 2k+1.Therefore, ∀ n ≥ 4, n! > 2n. |
| 4.4-5Prove: ∀ n ≥ 1, Let n=1 Observe: 12(1+1)=2Induction Suppose k ≥ 1, . (Goal: ) Observe that:  = k2(k+1)+3(k+1)2-(k+1) = (k+1)[k2+3(k+1)-1] = k+1[k2+3k+2] = (k+1)(k+1)(k+2) = (k+1)2((k+1)+1) That is: .Therefore, ∀ n ≥ 1,  |

Section 4.1: Odd exercises 1-27, 29, and 51 (Note that 25 is located at the top of this paper).

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| 4.1-1Show: 10! 10\*9\*8\*7\*6\*5\*4\*3\*2 90\*56\*30\*12\*2 5040\*360\*2 1814400\*2**3628800** |
| 4.1-3Show:  |
| 4.1-5Show:  |
| 4.1-7Prove: ∀ n, k ∈ Z, 1 ≤ k ≤ n,  Observe,  |
| 4.1-9True or False: ∀ n ∈ N, (n2)!=(n!)2Let n =3. Observe: (32)!=362,880 (3!)2=36Therefore false, ∃ n ∈ N such that (n2)!≠(n!)2. |

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| 4.1-11True or False: ∀ even n ∈ N,. Let n=4 Observe:  That is, 2 ≠ 12.Therefore false, ∃ even n ∈ N such that . |
| 4.1-13What are the first four terms of: ∀ n≥ 0, Sn=4-2n? S0=4-2(0)= 4 S1=4-2(1)= 2 S2=4-2(2)= 0 S3=4-2(3)= -2 |
| 4.1-15What are the first four terms of: ∀ n ≥ 3, Sn=? S3= 6 S4= 12 S5= 40 S6= 180 |
| 4.1-17What are the first four terms of: ∀n ≥2, Sn=3+2n S2=3+2(2)= 7 S3=3+2(3)= 9 S4=3+2(4)= 11 S5=3+2(5)= 13 |

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| 4.1-19What is the 15th prime number? 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47 S15=47 |
| 4.1-21What is the closed formula for the sequence: 2,4,6,8,10? ∀ n ≥ 1, Sn=2n S1=2(1)=2 S2=2(2)=4 S3=2(3)=6 S4=2(4)=8 S5=2(5)=10 |
| 4.1-23What is the closed formula for the sequence: 3, 6, 12, 24, 48. ∀ n ≥ 0, Sn=3(2n) S0=3(20)=3 S1=3(21)=6 S2=3(22)=12 S3=3(23)=24 S4=3(24)=48 |

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| 4.1-27What is the closed formula for the sequence: 1, ½, 1/3, ¼, 1/5…? ∀n ≥1, Sn=1/n S1=1/1=1 S2=1/2 S3=1/3 S4=1/4 S5=1/5 |
| 4.1-29Use: ∀ n ≥ 1, P0=x, Pn=Pn-1+(iPn-1)

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| A: Given P0=$6000 and i=0.03P1=6000+(6000\*0.03)=$6,180.00P2=6180+(6180\*0.03)=$6,365.40 | B: Given P0=$1000 and i=b P1=1000+(1000b) P2= P1+( P1b)Total Interest Earned = P2- P0Given i=.01 P1=$1,010 P2=$1,020.10Total Interest Earned=$20.10Given i=.02 P1=$1,020 P2=$1,040.40Total Interest Earned=$40.40Note that $40.40=2.01($20.10)As such, it is over twice as much interest earned. | C: This sequence∀ n ≥ 1, P0=x, Pn=Pn-1+(iPn-1)Is a recursive arithmetic sequence in Pn-1+(a), and a recursive geometric sequence in (iPn-1). As such, it is a hybrid sequence. |

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| 4.1-51Use S0=0, S1=D, S2=D(1+i)+D,…

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| A: Given D=1000, i= 0.05S2=1000(1+0.05)+1000=$2050S3=2050(1+0.05)+1000=$3152.50 | B: S10=$1200.61, i=0.04, D=$100S11=1200.61(1+0.04)+100=$1348.63 | C: Recursive Formula∀ n ≥ 2, D0=0, D1=x, Dn=Dn-1(1+i)+D1 |

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Section 4.2: Odd exercises 3-13, 15, and 39.

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| 4.2-3Solve and show the sigma notation for: 1+8+27+64+…+100013+23+33+43+…+103= |
| 4.2-5Solve and show the sigma notation for: 1+4+9+16…+1,048,576.12+22+32+42…+1,0242= |
| 4.2-7Solve and show the sigma notation for: -2+4-8+16-…-512-21+-22+-23+-24…+-29= |
| 4.2-9Solve and show the sigma notation for: 12+27+48+75+…+3n23(2)2+3(3)2+3(4)2+3(5)2+…+3n2= |
| 4.2-11Solve and show the sigma notation for: 41+42+43+…4n41+42+43+44+…+4n= |
| 4.2-13Solve and show the sigma notation for: 9-27+81-243+…+(-3)n(-3)2+(-3)3+(-3)4+(-3)5+….+(-3)n= |
| 4.2-39Show:  |

Section 4.3: Odd exercises 1-13.

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| 4.3-1Prove: ∀ n ≥ 3, n2+1 ≥ 3n.Let n = 3 Observe: (32+1 ≥ 3(3)) = 10 ≥ 9Induction Suppose k ≥ 3, k2+1 ≥ 3k. (Goal: (k+1)2+1≥3(k+1)) Observe that: (k+1)2+1= k2+2k+1+1 = (k2+1)(2k+1), [Note that 2k+1 is added to the primary equation] ≥ 3 + (2k+1) ≥ 3k+3 =3(k+1) That is, (k+1)2+1 ≥ 3(k+1)Therefore, ∀ n ≥ 3, n2+1 ≥ 3n. |
| 4.3-3Prove: ∀ n ≥ 3, n2 ≥ 2n + 1.Let n = 3 Observe: (32 ≥ 2(3)+1) = 9 ≥ 7Induction Suppose k ≥ 3, k2 ≥ 2k+1. (Goal: (k+1)2 ≥ 2(k+1)+1) Observe that: (k+1)2= k2 + (2k+1) [Note that 2k+1 is added to the primary equation.] ≥ 2k +1 + (2k+1) = 2k+(2k+2) ≥ 2k+3 = 2(k+1)+1 That is, (k+1)2 ≥ 2(k+1) +1.Therefore, ∀ n ≥ 3, n2 ≥ 2n + 1. |
| 4.3-5Prove: ∀n ≥ 4, 2n ≥ n2.Let n = 5. Observe: (25 ≥ 52) = 32 ≥ 25.Induction Suppose k ≥ 4, 2k ≥ k2. (Goal: 2k+1 ≥ (k+1)2) Observe that: 2k+1 = (2k)2 [Note that 2 is added to the initial equation] ≥ 2k2 ≥ k2 + (2k+1) = (k+1)2 That is, 2k+1 ≥ (k+1)2.Therefore, ∀n ≥ 4, 2n ≥ n2. |
| 4.3-7Prove: ∀ n ≥ 4, n! ≥ n2.Let n = 5. Observe: (5! ≥ 52) = 120 ≥ 25.Induction Suppose k ≥ 4, k! ≥ k2. (Goal: (k+1)! ≥ (k+1)2). Observe that: (k+1)! = (k+1)k! ≥ (k+1)k2 ≥ (k+1)2. That is, (k+1)! ≥ (k+1)2.Therefore, ∀ n ≥ 4, n! ≥ n2. |

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| 4.3-11Prove: ∀ n ≥ 0, 3 | (4n-1).Let n = 3. Observe, 43-1= 63, 3|63.Induction Suppose, k ≥ 0, 3|(4k-1). So, 4k-1=3m where m ∈ Z. (Goal: 3|(4k+1-1)) Observe that: 4k+1-1 is equivalent to 4k+1≡1(mod 3) Also, 4k+1=4(4k). 4≡1(mod 3). That is, 3|(4k+1-1).Therefore, ∀ n ≥ 0, 3 | (4n-1). |
| 4.3-13Prove: ∀ n ≥ 0, 4 | (6n-2n)Let n = 3. Observe: 63-23= 208, 4 | 208.Suppose k ≥ 0, 4|(6k-2k). So, 6k-2k= 4m, m ∈ Z. Observe that: 6k+1-2k+1 = 6(6k)-2(2k) = 4(6k)+2(6k-2k) = 4(6k)+2(4m) = 24k+8m = 4(6k+8m) Specifically, 4|4(6k+8m) That is, 4 | (6k-2k).Therefore, ∀ n ≥ 0, 4 | (6n-2n). |

Section 4.4: Odd exercises 5-15 and 21 (Note that question 5 is located at the top of this paper).

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| 4.4-7Prove: ∀ n ≥ 1, 1+5+9+…+ (4n-3)=n(2n-1)..Let n = 1, 1(2(1)-1)=1.Induction Suppose, k ≥ 1, (Goal:  Observe that:  = (4(2k-1)+4k+1) = 2k2-k+4k+1 = 2k2+3k+1 = (2k+1)(k+1) = k+1(2(k+1)-1) That is, Therefore,  |
| 4.4-9Show: ∀ n ≥ 1, 3+5+7+…+(2n+1)=n(n+2)Prove:  Let n =2, 2(2+2)=8Induction Suppose k ≥ 1,  Observe that:  = k(k+2)+(2k+3) = k2+4k+3 =(k+1)((k+1)+2) That is, Therefore,  |
| 4.4-11Show: ∀ n ≥ 0, 1+2+4+8+…+2n=2n+1-1.Prove: ∀ n ≥ 0,  Let n=0, 20+1-1=1Induction Suppose k ≥ 0,  Observe that:  = (2k+1-1)+(2k+1) = (2(2k)-1)+(2(2k) = 2[2k+2k-1] = 2(k+1)+1-1 That is, Therefore, ∀ n ≥ 0,  |
| 4.4-13Prove: ∀ n ≥ 2, Let n = 2, (2-1)22+1=8.Induction Suppose k ≥ 2, . Observe that:  = ((k-1)2k+1)+((k+1)2k+1) = ((k+1)-1)2(k+1)+1 That is.Therefore, ∀ n ≥ 2,  |

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| 4.4-15Prove: ∀ n ≥ 1, Let n = 1, (12-21+3)21+1-6=2Induction Suppose k ≥ 1,  (Goal: ) Observe that:  = = That is Therefore, ∀ n ≥ 1, . |
| 4.4-21Prove: ∀ n ≥ 1, .Let n =1, 1/(1+1)=1/2.Induction Suppose k ≥ 1, . (Goal: ) Observe that: = =  =  =  That is.Therefore, ∀ n ≥ 1, . |

Reference

Ferland, K. (2009). *Discrete Mathematics.* Boston: Houghton Mifflin Company.