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Week 4 Application: Indexed by Integers

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Graded problems: 4.1-4, 4.1-25, 4.2-8, 4.3-9, and 4.4-5.

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| 4.1-4  Show: |
| 4.1-25  What is the closed sequence for: 1, -3, 5, -7, 9…?  ∀ n ≥ 0, Sn = -1n(1+2n)  S0=-10(1+2(0))= 1  S1=-11(1+2(1))= -3  S2=-12(1+2(2))= 5  S3=-13(1+2(3))= -7  S4=-14(1+2(4))= 9 |
| 4.2-8  “The sum of the odd integers from 5 to 1001” (Ferland, 2009, p. 182).  Solve and show the sigma notation for: 1+3+5+7+…+1001. |

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| 4.3-9  Prove: ∀ n ≥ 4, n! > 2n.  Let n=5.  Observe: (5!>25)=120>32.  Induction  Suppose k ≥ 4, k! > 2k. (Goal: (k+1)! > 2k+1).  Observe that: (k+1)! = (k+1)k!  > 2k(k+1)  > 2k(2)  = 2k+1.  That is, (k+1)! > 2k+1.  Therefore, ∀ n ≥ 4, n! > 2n. |
| 4.4-5  Prove: ∀ n ≥ 1,  Let n=1  Observe: 12(1+1)=2  Induction  Suppose k ≥ 1, . (Goal: )  Observe that:  = k2(k+1)+3(k+1)2-(k+1)  = (k+1)[k2+3(k+1)-1]  = k+1[k2+3k+2]  = (k+1)(k+1)(k+2)  = (k+1)2((k+1)+1)  That is: .  Therefore, ∀ n ≥ 1, |

Section 4.1: Odd exercises 1-27, 29, and 51 (Note that 25 is located at the top of this paper).

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| 4.1-1  Show: 10!  10\*9\*8\*7\*6\*5\*4\*3\*2  90\*56\*30\*12\*2  5040\*360\*2  1814400\*2  **3628800** |
| 4.1-3  Show: |
| 4.1-5  Show: |
| 4.1-7  Prove: ∀ n, k ∈ Z, 1 ≤ k ≤ n,  Observe, |
| 4.1-9  True or False: ∀ n ∈ N, (n2)!=(n!)2  Let n =3.  Observe: (32)!=362,880  (3!)2=36  Therefore false, ∃ n ∈ N such that (n2)!≠(n!)2. |

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| 4.1-11  True or False: ∀ even n ∈ N,.  Let n=4  Observe:  That is, 2 ≠ 12.  Therefore false, ∃ even n ∈ N such that . |
| 4.1-13  What are the first four terms of: ∀ n≥ 0, Sn=4-2n?  S0=4-2(0)= 4  S1=4-2(1)= 2  S2=4-2(2)= 0  S3=4-2(3)= -2 |
| 4.1-15  What are the first four terms of: ∀ n ≥ 3, Sn=?  S3= 6  S4= 12  S5= 40  S6= 180 |
| 4.1-17  What are the first four terms of: ∀n ≥2, Sn=3+2n  S2=3+2(2)= 7  S3=3+2(3)= 9  S4=3+2(4)= 11  S5=3+2(5)= 13 |

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| 4.1-19  What is the 15th prime number?  2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47  S15=47 |
| 4.1-21  What is the closed formula for the sequence: 2,4,6,8,10?  ∀ n ≥ 1, Sn=2n  S1=2(1)=2  S2=2(2)=4  S3=2(3)=6  S4=2(4)=8  S5=2(5)=10 |
| 4.1-23  What is the closed formula for the sequence: 3, 6, 12, 24, 48.  ∀ n ≥ 0, Sn=3(2n)  S0=3(20)=3  S1=3(21)=6  S2=3(22)=12  S3=3(23)=24  S4=3(24)=48 |

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| 4.1-27  What is the closed formula for the sequence: 1, ½, 1/3, ¼, 1/5…?  ∀n ≥1, Sn=1/n  S1=1/1=1  S2=1/2  S3=1/3  S4=1/4  S5=1/5 |
| 4.1-29  Use: ∀ n ≥ 1, P0=x, Pn=Pn-1+(iPn-1)   |  |  |  | | --- | --- | --- | | A: Given P0=$6000 and i=0.03  P1=6000+(6000\*0.03)=$6,180.00  P2=6180+(6180\*0.03)=$6,365.40 | B: Given P0=$1000 and i=b  P1=1000+(1000b)  P2= P1+( P1b)  Total Interest Earned = P2- P0  Given i=.01  P1=$1,010  P2=$1,020.10  Total Interest Earned=$20.10  Given i=.02  P1=$1,020  P2=$1,040.40  Total Interest Earned=$40.40  Note that $40.40=2.01($20.10)  As such, it is over twice as much interest earned. | C: This sequence  ∀ n ≥ 1, P0=x, Pn=Pn-1+(iPn-1)  Is a recursive arithmetic sequence in Pn-1+(a), and a recursive geometric sequence in (iPn-1). As such, it is a hybrid sequence. | |

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| 4.1-51  Use S0=0, S1=D, S2=D(1+i)+D,…   |  |  |  | | --- | --- | --- | | A: Given D=1000, i= 0.05  S2=1000(1+0.05)+1000=$2050  S3=2050(1+0.05)+1000=$3152.50 | B: S10=$1200.61, i=0.04, D=$100  S11=1200.61(1+0.04)+100=$1348.63 | C: Recursive Formula ∀ n ≥ 2, D0=0, D1=x,  Dn=Dn-1(1+i)+D1 | |

Section 4.2: Odd exercises 3-13, 15, and 39.

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| 4.2-3  Solve and show the sigma notation for: 1+8+27+64+…+1000  13+23+33+43+…+103= |
| 4.2-5  Solve and show the sigma notation for: 1+4+9+16…+1,048,576.  12+22+32+42…+1,0242= |
| 4.2-7  Solve and show the sigma notation for: -2+4-8+16-…-512  -21+-22+-23+-24…+-29= |
| 4.2-9  Solve and show the sigma notation for: 12+27+48+75+…+3n2  3(2)2+3(3)2+3(4)2+3(5)2+…+3n2= |
| 4.2-11  Solve and show the sigma notation for: 41+42+43+…4n  41+42+43+44+…+4n= |
| 4.2-13  Solve and show the sigma notation for: 9-27+81-243+…+(-3)n  (-3)2+(-3)3+(-3)4+(-3)5+….+(-3)n= |
| 4.2-39  Show: |

Section 4.3: Odd exercises 1-13.

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| 4.3-1  Prove: ∀ n ≥ 3, n2+1 ≥ 3n.  Let n = 3  Observe: (32+1 ≥ 3(3)) = 10 ≥ 9  Induction  Suppose k ≥ 3, k2+1 ≥ 3k. (Goal: (k+1)2+1≥3(k+1))  Observe that: (k+1)2+1= k2+2k+1+1  = (k2+1)(2k+1), [Note that 2k+1 is added to the primary equation]  ≥ 3 + (2k+1)  ≥ 3k+3  =3(k+1)  That is, (k+1)2+1 ≥ 3(k+1)  Therefore, ∀ n ≥ 3, n2+1 ≥ 3n. |
| 4.3-3  Prove: ∀ n ≥ 3, n2 ≥ 2n + 1.  Let n = 3  Observe: (32 ≥ 2(3)+1) = 9 ≥ 7  Induction  Suppose k ≥ 3, k2 ≥ 2k+1. (Goal: (k+1)2 ≥ 2(k+1)+1)  Observe that: (k+1)2= k2 + (2k+1) [Note that 2k+1 is added to the primary equation.]  ≥ 2k +1 + (2k+1)  = 2k+(2k+2)  ≥ 2k+3  = 2(k+1)+1  That is, (k+1)2 ≥ 2(k+1) +1.  Therefore, ∀ n ≥ 3, n2 ≥ 2n + 1. |
| 4.3-5  Prove: ∀n ≥ 4, 2n ≥ n2.  Let n = 5.  Observe: (25 ≥ 52) = 32 ≥ 25.  Induction  Suppose k ≥ 4, 2k ≥ k2. (Goal: 2k+1 ≥ (k+1)2)  Observe that: 2k+1 = (2k)2 [Note that 2 is added to the initial equation]  ≥ 2k2  ≥ k2 + (2k+1)  = (k+1)2  That is, 2k+1 ≥ (k+1)2.  Therefore, ∀n ≥ 4, 2n ≥ n2. |
| 4.3-7  Prove: ∀ n ≥ 4, n! ≥ n2.  Let n = 5.  Observe: (5! ≥ 52) = 120 ≥ 25.  Induction  Suppose k ≥ 4, k! ≥ k2. (Goal: (k+1)! ≥ (k+1)2).  Observe that: (k+1)! = (k+1)k!  ≥ (k+1)k2  ≥ (k+1)2.  That is, (k+1)! ≥ (k+1)2.  Therefore, ∀ n ≥ 4, n! ≥ n2. |

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| 4.3-11  Prove: ∀ n ≥ 0, 3 | (4n-1).  Let n = 3.  Observe, 43-1= 63, 3|63.  Induction  Suppose, k ≥ 0, 3|(4k-1).  So, 4k-1=3m where m ∈ Z. (Goal: 3|(4k+1-1))  Observe that: 4k+1-1 is equivalent to 4k+1≡1(mod 3)  Also, 4k+1=4(4k).  4≡1(mod 3).  That is, 3|(4k+1-1).  Therefore, ∀ n ≥ 0, 3 | (4n-1). |
| 4.3-13  Prove: ∀ n ≥ 0, 4 | (6n-2n)  Let n = 3.  Observe: 63-23= 208, 4 | 208.  Suppose k ≥ 0, 4|(6k-2k).  So, 6k-2k= 4m, m ∈ Z.  Observe that: 6k+1-2k+1 = 6(6k)-2(2k)  = 4(6k)+2(6k-2k)  = 4(6k)+2(4m)  = 24k+8m  = 4(6k+8m)  Specifically, 4|4(6k+8m)  That is, 4 | (6k-2k).  Therefore, ∀ n ≥ 0, 4 | (6n-2n). |

Section 4.4: Odd exercises 5-15 and 21 (Note that question 5 is located at the top of this paper).

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| 4.4-7  Prove: ∀ n ≥ 1, 1+5+9+…+ (4n-3)=n(2n-1).  .  Let n = 1, 1(2(1)-1)=1.  Induction  Suppose, k ≥ 1, (Goal:  Observe that:  = (4(2k-1)+4k+1)  = 2k2-k+4k+1  = 2k2+3k+1  = (2k+1)(k+1)  = k+1(2(k+1)-1)  That is,  Therefore, |
| 4.4-9  Show: ∀ n ≥ 1, 3+5+7+…+(2n+1)=n(n+2)  Prove:  Let n =2, 2(2+2)=8  Induction  Suppose k ≥ 1,  Observe that:  = k(k+2)+(2k+3)  = k2+4k+3  =(k+1)((k+1)+2)  That is,  Therefore, |
| 4.4-11  Show: ∀ n ≥ 0, 1+2+4+8+…+2n=2n+1-1.  Prove: ∀ n ≥ 0,  Let n=0, 20+1-1=1  Induction  Suppose k ≥ 0,  Observe that:  = (2k+1-1)+(2k+1)  = (2(2k)-1)+(2(2k)  = 2[2k+2k-1]  = 2(k+1)+1-1  That is,  Therefore, ∀ n ≥ 0, |
| 4.4-13  Prove: ∀ n ≥ 2,  Let n = 2, (2-1)22+1=8.  Induction  Suppose k ≥ 2, .  Observe that:  = ((k-1)2k+1)+((k+1)2k+1)  = ((k+1)-1)2(k+1)+1  That is.  Therefore, ∀ n ≥ 2, |

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| 4.4-15  Prove: ∀ n ≥ 1,  Let n = 1, (12-21+3)21+1-6=2  Induction  Suppose k ≥ 1,  (Goal: )  Observe that:  =  =  That is  Therefore, ∀ n ≥ 1, . |
| 4.4-21  Prove: ∀ n ≥ 1, .  Let n =1, 1/(1+1)=1/2.  Induction  Suppose k ≥ 1, . (Goal: )  Observe that: =  =  =  =  That is.  Therefore, ∀ n ≥ 1, . |

Reference

Ferland, K. (2009). *Discrete Mathematics.* Boston: Houghton Mifflin Company.