Running Head: ELEMENTARY NUMBER THEORY

Week 3 Application: Elementary Number Theory

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Week 3 Application: Elementary Number Theory

Questions: 3.1-1, 3.2-15, 3.4-2, 3.5-4, 3.6-1

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| 3.1-1  Let m and n be arbitrary integers.  Consider the product of an even m and an odd n is always even.  Let m=2k and n=2l+1  That is, 2k(2l+1)= 2(2kl+k)=mn.  Note that 2(2kl+k) is even.  Therefore, the product of an even integer and an odd integer is always even. |
| 3.2-15  “Compute 73 div 10 and 73 mod 10” (Ferland, 2009, p. 124).  Let 73, 10 ∈ Z.  Observe:    That is, 73=10(7)+3  Such that, n = 73, d = 10, q = 7, r = 3  Therefore, 73÷10. |
| 3.4-2  Prove:  Observe that .  23 div 3 = 7  3 mod 3 = 2  Note that 3 is a prime element of Z and is not divisible by 2 or 5.  Therefore, |

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| 3.5-4  Show: .  Contradiction: Observe  That is,  **Specifically, gcd(a,b) = 1**.  Thus,  So, a2 =pb2.  That is, a mod 2 = 0.  Thus a = pk where k ∈ Z. It follows pb2=a2=p2k2.  So, b2=pk2.  Since b2 mod 2 = 0, then b mod 2 =0.  **Note, gcd(a,b)=2.** This is a contradiction.  Therefore |
| 3.6-1  True or False: 55 ≡ 15(mod 10)  Observe that 15|(55-10)  15(3)=45  45+10=55  Therefore, 55 ≡ 15(mod 10) is a true assertion. |

Section 3.1: Odd exercises 1-13 (Note that question 1 is at the top of this paper.

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| 3.1-3  Consider if n2 is even then n is even.  Let n ∈ Z and n=2k.  That is n2=n(n)=2k(2k)=4k2  Note that 4k2 is even.  Therefore, if n2 is even then n is even. |
| 3.1-5  Consider for every even integer, n, (-1)n=1.  Let n ∈ Z and n = 2k.  Such that, (-1)2k=1.  Observe that, .  Note.  Therefore, for every even integer, n, (-1)n=1. |
| 3.1-7  What is the status of a Television after the power button on its remote is pressed 50 times, starting from on?  Let on=1 and off = -1.  Specifically, on = (-1)2 .  (Which is the same formula 3.1-5, for every even integer, n, (-1)n=1 .)  Thus, the status is (-1)50=1.  Therefore, the Television will be on after the power button is pressed 50 times. |

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| 3.1-9  Consider a|0.  Let a ∈ Z.  Note that 0=ak.  That is, 0 ∈ a or 0 ∈ k.  In either case ak=0.  Thus, a|0. |
| 3.1-11  Consider if a|1, then a = ± 1.  That is 1=ak.  Note that when a=1, k=1.  Also, when a=-1, k=-1.  -1(-1)=-12=1, and 1(1)=12=1.  Therefore, if a|1, then a = ± 1. |

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| 3.1-13  (Part A): Consider, (a-1)|(a2-1) where a ∈ Z.  Observe (a-1)|(a2-1) =  (a-1)|(a+1)(a-1) =  (a+1)  (Part B):   |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | |  | (a-1)(a-1) | | |  |  | | a2 |  |  |  |  |  | |  |  |  |  | |  |  |  |  | |  |  |  |  |  | |  | (a-1) | | |  |  | |

Section 3.2: Odd exercises 11-19 (Note that question 15 is at the top of this paper).

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| 3.2-11  “Find the quotient and remainder resulting from the integer division problems” (Ferland, 2009, p. 124).   |  |  | | --- | --- | | A: 127÷12  Let n, d, q, r ∈ Z.  If n = 127 and d=12, then q = 10 and r = 7  That is, 127=12(10)+7  Such that, 127 div 12 = 10 and 127 mod 12 = 7 | B: 216÷15  Let n, d, q, r ∈ Z.  If n = 216 and d = 15, then q = 14 and r = 6  That is, 216=15(14)+6  Such that, 216 div 15 = 14 and 216 mod 15 = 6 | |
| 3.2-13  “Verify the conclusions of the Division Algorithm for each of the following” (Ferland, 2009, p. 124).   |  |  | | --- | --- | | A: n = 45 and d = 7  Let n, d ∈ Z.  Note that 45 div 7 = 6 and 45 mod 7 = 3  That is, 45=7(6)+3  Such that, n = 45, d = 7, q = 6, and r = 3  Where q, r ∈ Z.  Therefore, 45÷7. | B: n=-37 and d = 4  Let n, d ∈ Z.  Note that -37 div 4 = -9 and -37 mod 4 = -1  That is, -37 = 4(-9)-1  Such that, n = -37, d = 4, q = -9, and r = -1  Where q, r ∈ Z.  Therefore, -37÷4 | |
| 3.2-17  Compute the following:   |  |  | | --- | --- | | A: 67 div 13 and 67 mod 13  Let 67, 13 ∈ Z.  Observe:    That is, 67=13(5)+2  Such that, n = 67, d = 13, q = 5, r = 2  Therefore, 67÷13. | B: -67 div 13 and -67 mod 13  Let -67, 13 ∈ Z.  Observe:    That is, -67=13(-5)-2  Such that, n = -67, d = 13, q = -5, r = -2  Therefore, -67÷13. | |

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| 3.2-19  Given 165 people who need to sit, and rows of 18 seats. How many rows are required?  Let 165, 18 ∈ Z.  Calculate: 165÷18= ⎡n⎤  165 div 18 = 9  165 mod 18 = 3  Note that there are still 3 people left standing.  ⎡n⎤ = ⎡9⎤ = 10  Therefore, 10 rows of seating are required. |

Section 3.4: Odd exercises 1-27

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| 3.4-1  Prove: .  Observe that  That is, 11 div 2 = 5.  11 mod 2 = 1.  Note that 2 is a divisor of itself.  Therefore, . |
| 3.4-3  Prove:  Observe that  That is, -67 div 5 = -13  -67 mod 5 = -2.  Note that 5 is a divisor of itself.  Therefore, |
| 3.4-5  Prove: 5.821 ∈ Q.  Observe that 5.821 =  That is, 5821 div 1000 = 5  5821 mod 1000 = 821  5821 = 1000(5)+821  Note: 1000 mod 2 = 0 and 1000 mod 5 = 0  Therefore, 5.821 ∈ Q. |

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| 3.4-7  Prove:  Observe that  That is, 311 div 99 = 3  311 mod 99 = 14  Such that, 311=99(3)+14  Note that 99 ∈ Z, 99 ≠ 0, and is repeating.  Therefore, |
| 3.4-9  Prove:  Observe that  That is, -713 div 165 = -4  -713 mod 165 = -53  Such that, -713 = 165(-4)-53  Note that 165 mod 5 = 0.  Therefore, |
| 3.4-11  Let r ∈ Q and n ∈ Z.  Prove: nr ∈ Q.  If .  Therefore, nr ∈ Q. |

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| 3.4-13  Let s, r ∈ Q.   |  |  | | --- | --- | | A: Prove –s ∈ Q.  If .  Therefore, –s ∈ Q. | B: Prove r – s ∈ Q.  Since  Then  Therefore, r – s ∈ Q. | |
| 3.4-15  Let r ∈ Q and n ∈ Z.  Prove: If n ≥ 0, then rn ∈ Q.  Note that rn = r(rm+n).  Observe that r(rm+1) ∈ Q.  Therefore, If n ≥ 0, then rn ∈ Q. |
| 3.4-17  Express in lowest term: .  Let 65, 39 ∈ Z.  gcd(65,39)=13  65 div 13 = 5  39 div 13 = 3  Therefore, . |

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| 3.4-19  Express in lowest term: .  Let -513, 72 ∈ Z.  gcd(-513,72)= 9  -513 div 9 = -57  72 div 9 = 8  Therefore, . |
| 3.4-21  Express in lowest term: 3.14  Note that 3.14 =  That is, 314 = 3(100)+14.  So gcd(314,100) = 2  314 div 2 = 157  100 div 2 = 50  Therefore, . |
| 3.4-23  Express in decimal: .  12 \* 10 = 120 **.**  25 \* 4 = 100 **4**  20 \* 10 = 200  25 \* 8 = 200 **8**  Therefore, .48 |

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| 3.4-25  Express in decimal.  3 \*10 = 30 .  7 \* 4 = 28 4  2 \* 10 = 20  7 \* 2 = 14 2  6 \* 10 = 60  7 \* 8 = 56 8  4 \* 10 = 40  7 \* 5 = 35 5  5 \* 10 = 50  7 \* 7 = 49 7  1 \* 10 = 10  7 \* 1 = 7 1  3 \* 10 = 30, Note this is the first repeat.  Therefore,. |
| 3.4-27  Express in decimal:  8 \* 10 = 80 **.**  15 \* 5 = 75 **5**  5 \* 10 = 50  15 \* 3 = 45 **3**  5 \* 10 = 50, Note this is the first repeat, and will continue to repeat.  Therefore, |

Section 3.5: Exercises 1-7.

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| 3.5-1  Show:  Contradiction: Observe  That is,  **Specifically, gcd(a,b) = 1**.  Thus, So, a2 =3b2.  That is, a mod 2 = 0.  Thus a = 3k where k ∈ Z. It follows 3b2=a2=9k2.  So, b2=3k2.  Since b2 mod 2 = 0, then b mod 2 =0.  **Note, gcd(a,b)=2.** This is a contradiction.  Therefore |
| 3.5-3  Show:  Contradiction: Observe  That is,  **Specifically, gcd(a,b) = 1**.  Thus,  So, a2 =13b2.  That is, a mod 2 = 0.  Thus a = 13k where k ∈ Z. It follows 13b2=a2=169k2.  So, b2=13k2.  Since b2 mod 2 = 0, then b mod 2 =0.  **Note, gcd(a,b)=2.** This is a contradiction.  Therefore |
| 3.5-7  Show:  Contradiction: Observe  That is,  **Specifically, gcd(a,b) = 1**.  Thus,  So, a3 =7b3.  Since 7|a3, 7|a. That is, a = 7c  Thus a = 7c where c ∈ Z. It follows 7b3=a3=343k3.  So, b3=7k3.  Since 7|b3, then 7|b.  **Note, gcd(a,b)=7.** This is a contradiction.  Therefore |

Section 3.6: Odd exercises 1-7, 39, 41, and 47 (Note that question 1 is located at the top of this paper).

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| 3.6-3  True or False: -7 ≡ 21 (mod 6).  Observe: -7|(21-6) is a false statement  gcd(15,-7)=1  Therefore, -7 ≡ 21 (mod 6) is a false assertion. |
| 3.6-5  What day was it on Jan 8, 1987 if it was Monday Oct 19, 1987?  There are 283 days between Jan 8, 1987 and Oct 19, 1987 (assuming this is not a leap year).  There are 7 days in a week.  283 mod 7 = 3.  Given: 0 1 2  **3** 4 5 6  M T W **Th** F S Su  Therefore, it was a Thursday on Jan 8, 1987. |
| 3.6-7  What time is it 279 hours from 6 A.M.?  279 hours + 6 hours = 285 hours  285 mod 24 = 21  21 mod 12 = 9  285 div 24 = 11  Therefore, it is 9 P.M. 11 days later. |

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| 3.6 – 39  Given the linear cipher function f=(2x+8) encrypt the message “DISCRETE MATH” using a modulus of 27 and the English alphabet with a space character for 0.  D 2(4)+8 mod 27=16 P 2(0)+8 mod 27= 8 H  I 2(9)+8 mod 27= 26 Z M 2(13)+8 mod 27= 7 G  S 2(19)+8 mod 27= 19 S A 2(1)+8 mod 27= 10 J  C 2(3)+8 mod 27= 14 N T 2(x)+8 mod 27= 21 U  R 2(18)+8 mod 27=17 Q H 2(8)+8 mod 27= 24 X  E 2(5)+8 mod 27= 18 R  T 2(20)+8 mod 27= 21 U  E 2(5)+8 mod 27= 18 R  Encrypted: “PZSNQRURHGJUX” |
| 3.6-41  Decrypt “QPVOZ” using x = 14 (y – 13) mod 27.  Q 14 (17 – 13) mod 27 = 2 B  P 14 (16 – 13) mod 27 = 15 O  V 14 (22 – 13) mod 27 = 18 R  O 14 (15 – 13) mod 27 = 1 A  Z 14 (y – 13) mod 27 = 20 T  Decrypted: “BORAT” |
| 3.6-47  Use: x7(mod 55) = y to encrypt. Use: y3(mod 55)=x to decrypt.   |  |  | | --- | --- | | A.  American Express, 87(mod 55) = 2 Encrypted | B.  Visa, 497(mod 55) = 14 Encrypted | | C.  Encrypted 12, 123(mod 55)= 23 Discover | D.  Encrypted 35, 353(mod 55)=30 MasterCard | |

Reference

Ferland, K. (2009). *Discrete Mathematics.* Boston: Houghton Mifflin Company.