Running Head: BASIC PROOF WRITING

Week 2 Application: Basic Proof Writing

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Week 2 Application: Basic Proof Writing

Week 2 Graded Problems: 2.1-8, 2.2-34, 2.3-3, 2.4-1, 2.5-11

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| 2.1-8  “Show: ∃ m, n∈Z such that 3m+5n=11” (Ferland, 2009, p. 73).  Let m, n ∈ Z  Consider m=2 and n=1, observe that  3(2)+5(1)=6+5=11  Therefore, ∃ m, n∈Z such that 3m+5n=11 |
| 2.2-34  “Show: IF A ⊆ B and A ⊆ C, then A ⊆ B∩C” (Ferland, 2009, p. 82).  Let A, B, and C be arbitrary sets.  Suppose A ⊆ B and A ⊆ C.  Using the absorption rule, since A∩B=A.  Continuing the absorption rule, since A∩C=A.  It follows that B∩C=A.  Hence A ⊆ B∩C. |
| 2.3-3  “Show: ∀ x ∈ R, x = 2x if and only if x = 0” (Ferland, 2009, p. 86).  Let ∀ x ∈ R.  Suppose x = 2x if and only if x = 0  So that x=2x ↔ 0 = 2x. That is x=0.  (2(0)=0)  Therefore, ∀ x ∈ R, x = 2x if and only if x = 0 |

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| 2.4-1  “Show: The interval (1,2) has no smallest element” (Ferland, 2009, p. 93).  Consider the interval (1,2) has no smallest element.  (Contradiction) Claim: The interval (1,2) has no largest element.  Let n ∈ (1,2), is a larger element of (1,2)  This is a contradiction.  Therefore the interval (1,2) has no smallest element. |
| 2.5-11  “Show: A Δ B ⊆ A ∪ B” (Ferland, 2009, p. 98).  Let A and B be arbitrary sets.  Consider A Δ B ⊆ A ∪ B.  Case 1  → Let x ∈ A  Since x ∈ A, and A Δ B, then x ∈ B  ← Let x ∈ B  Since x ∈ B, and A Δ B, then x ∈ A.  That is, x ∈ A Δ B.  Case 2  Let x ∈ A  Since x ∈ A, and A ∪ B, then x ∈ B.  That is, x ∈ A ∪ B.  Overall, x ∈ A Δ B and x ∈ A ∪ B.  Therefore, A Δ B ⊆ A ∪ B. |

Section 2.1: Odd exercises 1-21 (pp. 73-74)

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| 2.1-1  “Show that the points (-1, -8), (1, -2), and (2, 1) lie on a common line” (Ferland, 2009, p. 73).  Let S be the line given by the equation y=3x-5, observe that  -8=3(-1)-5  -2=3(1)-5 and  1=3(2)-5  Therefore, all of the points (-1, -8), (1, -2), and (2, 1) lie on the same common line, S. |
| 2.1-3  “Show: There is a set A such that {1,2,3,4}\A={1,3}” (Ferland, 2009, p. 73).  Consider {1,2,3,4}\A={1,3}  Let the set A being equal to {2,4},  observe that {1,2,3,4}\{2,4}={1,3}  Therefore, there is a set A such that {1,2,3,4}\A={1,3}. |
| 2.1-5  “Show: There exist sets A and B such that (Ferland, 2009, p. 73).  Let A and B be arbitrary sets.  Consider the set of A being equal to {1} and the set of B being equal to {1}, observe that  and    Therefore, there does exist a set of A and B such that, . |

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| 2.1-7  “Show: ∃ n ∈ Z such that ” (Ferland, 2009, p. 73)  Let n ∈ Z  Consider  Let n= -3, observe that  and    Therefore, there exists an integer such that 10 taken to that integer’s power equals 0.001. |
| 2.1-9  “Show: ∃ m, n ∈ Z such that 9m+14n=1” (Ferland, 2009, p. 73)  Let m, n ∈ Z  Consider 9m+14n=1  Let m = -3, n = 2  Observe that 9(-3)+14(2)=-27+28=1  Therefore, ∃ m, n∈ Z such that 9m+14n=1 |
| 2.1-11  “Show: There exist sets A and B such that A\B = B\A” (Ferland, 2009, p. 73).  Consider where the set of A = Z and the set of B = Z, observe that  Z\Z=θ (the empty set)  Therefore, there exists sets A and B such that A\B = B\A. |
| 2.1-13  “Show: has two distinct real roots” (Ferland, 2009, p. 73).  Let x ∈ R  Consider that , observe that  = 1 and -1  Therefore, in order for x-1 and x+1 to factor out to 0, 1 and -1 must be the roots of. |
| 2.1-15  “Show: has no real roots” (Ferland, 2009, p. 73).  Let x ∈ R  Consider  Observe that = 3 and NAN  Therefore, has no real roots. |
| 2.1-17  “If P dollars is invested in a certain account, then the amount A in that account after t years is given by the formula A=P(1.075)^t” (Ferland, 2009, p. 73)  Consider A=P(1.075)^t  Let P =  Observe that 4851.94(1.075)^10  Therefore, there exists an amount of money which will give a return of $10,000, or greater, after ten years using A=P(1.075)^t. |
| 2.1-19  “Show: There is a set A such that A^2=A” (Ferland, 2009, p. 74).  Consider A^2=A  Let A = {} the empty set θ  Observe that {}^2={}  Therefore, there is a set A such that A^2=A |
| 2.1-21  “Disprove: The union of any two intervals is an interval” (Ferland, 2009, p. 74).  Let A, B, and C be arbitrary intervals.  Consider AB=C  Let A = (-2,-1), let B= (1,2)  Observe that (-2,-1)(1,2) = {(-2,-1),(1,2)} which is not an interval.  Therefore, there exists a union of two intervals with is not an interval. |

Section 2.2: Odd exercises 1-15 and 29-43 (pp. 81-82)

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| 2.2-1  “Show:∀ x ∈ R+, -x ∈ R-” (Ferland, 2009, p. 81).  Consider ∀ x ∈ R+, -x ∈ R-  Let x ∈ R+  So x ∈ R and x > 0.  Multiplication by -1 gives that –x < 0. (-1x = -x < 0)  Therefore, since –x ∈ R and x < 0, we have –x ∈ R- |
| 2.2-3  “Show: ∀ x ∈ R, if x ∈ (2,4), then 2x ∈ (4,8)” (Ferland, 2009, p. 81).  Consider ∀ x ∈ R, if x ∈ (2,4), then 2x ∈ (4,8)  Let x ∈ R. Also, let x ∈ (2,4). In essence{x: 2 <x < 4}.  Given 2(2) < 2x < 4(2) = 4 < 2x < 8.  Therefore, 2x ∈ (4,8) |
| 2.2-5  “Prove or Disprove: ∀ x ∈ R+, ” (Ferland, 2009, p. 81).  Consider ∀ x ∈ R+,  Let x = .5  Where x =  Therefore, ∃ x ∈ R+ such that . |
| 2.2-7  “Prove or Disprove: ∀ x ∈ R, if x < 2, then x2 < 4” (Ferland, 2009, p. 81).  Consider ∀ x ∈ R, if x < 2, then x2 < 4  Let x = -4, note that -4 ∈ R and -4 < 2.  That is, -42= 16 > 4  Therefore, ∃ x ∈ R such that x2 > 4. |

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| 2.2-9  “Show:∀ x ∈ R, if x < -2, then x2 > 4” (Ferland, 2009, p. 81).  Consider ∀ x ∈ R, if x < -2, then x2 > 4  Let x ∈ R  Observe that –x\*-x= x2  Note that -22=4  Therefore, ∀ x ∈ R, if x<-2, then x2>4. |
| 2.2-11  Voltage (V) is the product of Current (I) and Resistance (R). Consider: IR = V, if R > 2 then I < 5.  Let R = 2, and V =10.  Observe that 10= 2I.  That is, I = 5.  Therefore, IR = V, if R > 2 then I < 5. |
| 2.2-13  Consider ∀ f ∈ R, if f is periodic, then f2 is periodic (f being a function)  Let f be a periodic real function  Observe that f(x+p) = f(x).  That is, [f(x+p]2=[f(x)]2.  Therefore, ∀ f ∈ R, if f is periodic, then f2 is periodic. |
| 2.2-15  Consider ∀ f ∈ R, if f is constant, then 2f is constant (f being a function).  Let f be a constant real function, such that ∀ x ∈R, f(x) = c ∈ R.  Then it follows that ∀ x ∈R, 2f(x) = 2c ∈ R.  Therefore, ∀ f ∈ R, if f is constant, then 2f is constant. |

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| 2.2-29  “Show: A ⊆ A ∪ B” (Ferland, 2009, p. 82).  Let A, B, and C be arbitrary sets.  Consider A ⊆ A ∪ B  Let x ∈ A.  If x ∈ A then x ∈ A or x ∈ B  Therefore, A ⊆ A ∪ B. |
| 2.2-31  “Show: If A ⊆ A ∩ B, then A ⊆ B” (Ferland, 2009, p. 82).  Let A and B be arbitrary sets.  Consider If A ⊆ A ∩ B, then A ⊆ B  Let x ∈ A.  Observe if x ∈ A then x ∈ A and x ∈ B. Specifically, x ∈ B.  Therefore, If A ⊆ A ∩ B, then A ⊆ B. |
| 2.2-33  “Show: If A ⊆ B, then A ∩ C ⊆ B ∩ C” (Ferland, 2009, p. 82).  Let A, B, and C be arbitrary sets.  Consider If A ⊆ B, then A ∩ C ⊆ B ∩ C  Assume A ⊆ B  Let x ∈ A ∩ C.  If x ∈ A and x ∈ C, and since A ⊆ B, x ∈ B.  Therefore, If A ⊆ B, then A ∩ C ⊆ B ∩ C |

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| 2.2-35  “Show: (A ∩ B) ∩ C = A ∩ (B ∩ C)” (Ferland, 2009, p. 82).  Let A, B, and C be arbitrary sets.  Consider (A ∩ B) ∩ C = A ∩ (B ∩ C)  So, x ∈ (A ∩ B) ∩ C  ↔ ( x ∈ A ∧ x ∈ B) ∧ x ∈ C  ↔ x ∈ A ∧( x ∈ B ∧ x ∈ C)  ↔ x ∈ A ∩ (B ∩ C)  ∴ (A ∩ B) ∩ C = A ∩ (B ∩ C) |
| 2.2-37  “Show: A ∩ B = B ∩ A” (Ferland, 2009, p. 82).  Let A and B be arbitrary sets.  Consider A ∩ B = B ∩ A  So, x ∈ A ∩ B  ↔ x ∈ A ∧ x ∈ B  ↔ x ∈ B ∧ x ∈ A  ↔ x ∈ B ∩ A  ∴ A ∩ B = B ∩ A |
| 2.2-39  “Prove or Disprove: ” (Ferland, 2009, p. 82).  Let A and B be arbitrary sets.  Consider.  Assume A = {1} and B = {2}      ∴ |

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| 2.2-41  Show: A∪(B∩C) = (A∪B)∩(A∪C)  Let A, B, and C be arbitrary sets.  Consider A∪(B∩C) = (A∪B)∩(A∪C)  So, x ∈ A∪(B∩C)  ↔ x ∈ A ∧ ( x ∈B ∧ x ∈C)  ↔ (x ∈ A ∨ x ∈B) ∧ ( x ∈ A ∨ x ∈C)  ↔ x ∈ (A∪B)∩(A∪C)  ∴ A∪(B∩C) = (A∪B)∩(A∪C) |
| 2.2-43  Let A, B, and C be arbitrary sets.  Consider (A∩B)c = Ac ∪ Bc  So, x ∈ (A∩B)c  ↔ ¬ (x ∈ (A∩B))  ↔ ¬ (x ∈ A ∧ x ∈ B)  ↔ ¬ (x ∈ A) ∨ ¬ (x ∈ B)  ↔ x ∈ Ac ∨ x ∈ Bc  ↔ x ∈ Ac ∪ Bc  ∴ (A∩B)c = Ac ∪ Bc |

Section 2.3: Odd exercises 1-9 (p. 86) (Note: question 3 is located at the top of this paper)

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| 2.3-1  “Show: ∀ x ∈ R, x ∈ R- if and only if –x ∈ R+” (Ferland, 2009, p. 86).  Consider ∀ x ∈ R, x ∈ R- if and only if –x ∈ R+  (→) Let x ∈ R-, that is, x <0.  -1x > 0, in essence –x>0.  So, -x ∈ R+  (←) Let –x ∈ R+, that is, -x >0.  -1(-x) < 0, in essence x < 0.  So, x ∈ R-  Therefore, ∀ x ∈ R, x ∈ R- if and only if –x ∈ R+ |
| 2.3-5  “Let x ∈ R. Show: x3 > 0 if and only if x > 0” (Ferland, 2009, p. 86).  Consider x3 > 0 iff x > 0.  Let x ∈ R  -x\*-x\*-x=-x3<0, note that –x<0.  x\*x\*x=x3>0, note that x>0.  Therefore, ∀ x ∈ R, x3 > 0 iff x > 0 |
| 2.3-7  “Show: ∀ x ∈ R, 4-x < 2 if and only if x > 2” (Ferland, 2009, p. 86).  Consider 4-x < 2 if and only if x > 2.  Let x ∈ R  4 -x < 2 ↔ -x < -2 ↔ x > 2  ∴ ∀ x ∈ R, 4-x < 2 iff x > 2 |

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| 2.3-9  “Show: ∀ x ∈ R, x4 – 16 = 0 if and only if x2 – 4 = 0” (Ferland, 2009, p. 86)”  Consider x4 – 16 = 0 iff x2 – 4 = 0.  (→) Let x ∈ R  (x2 – 4)( x2 – 4)= x4 – 16  If x2 – 4 = 0 then 0\*0=0, so x4 – 16=0  (←)Let x ∈ R  (x2 – 4)2= x4 – 16  If x4 – 16=0 then (0)2=0, so x2 – 4=0  ∴ ∀ x ∈ R, x4 – 16 = 0 iff x2 – 4 = 0 |

Section 2.4: Odd exercises 1-9 (p. 93) (Note: question1 is located at the top of this paper)

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| 2.4-3  “Show: N has no largest element” (Ferland, 2009, p. 93).  Consider N has no largest element.  Let x ∈ N.  Observe that (x + 1) ∈ N  Thus the statement N has no largest element is a contradiction. |
| 2.4-5  “Show: For any set A, A ∩ θ = θ” (Ferland, 2009, p. 93)  Consider for any set A, A ∩ θ = θ.  (Contradiction) Claim: A ∩ θ ≠ θ  That is x ∈ A and x ∈ θ.  This is a contradiction, x cannot exist in θ.  Therefore, for any set A, A ∩ θ = θ. |
| 2.4-7  “Show: The interval (0,1] is infinite” (Ferland, 2009, p. 93).  Consider the interval (0,1] is infinite.  (Contradiction) Claim: the interval (0,1] is finite.  Let n = |(0,1]| (Note that (0,1] is a range).  Observe that is too large.  This is a contradiction.  Therefore the interval (0,1] is infinite. |

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| 2.4-9  “Show: is infinite” (Ferland, 2009, p. 93).  Consider is infinite.  (Contradiction) Claim: is finite.  Let n = ||  Observe that {(n+1)2, (n+1)} is infinite.  This is a contradiction.  Therefore, is infinite. |

Section 2.5: Odd exercises 1-15 (p. 98) (Note: question 11 is located at the top of this paper)

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| 2.5-1  “Show: If A ⊆ B and C ⊆ D, then A ∪ C ⊆ B ∪ D” (Ferland, 2009, p. 98).  Let A, B, C, and D be arbitrary sets.  Consider if A ⊆ B and C ⊆ D, then A ∪ C ⊆ B ∪ D.  Let x ∈ A or x ∈ C, such that x ∈ A ∪ C.  Case 1  Since x ∈ A and A ⊆ B, then x ∈ B.  As x ∈ B and B ∪ D, then x ∈ B ∪ D.  Case 2  Since x ∈ C and C ⊆ D, then x ∈ D.  As x ∈ D and B ∪ D, then x ∈ B ∪ D.  Overall x ∈ in A ∪ C and x ∈ B ∪ D.  Therefore, if A ⊆ B and C ⊆ D, then A ∪ C ⊆ B ∪ D. |
| 2.5-3  “Show: If A ⊆ B, then A ∪ B = B” (Ferland, 2009, p. 98).  Let A and B be arbitrary elements.  Consider if A ⊆ B, then A ∪ B = B.  Let x ∈ A.  Since x ∈ A and A ⊆ B, then x ∈ B.  (→) Since x ∈ B and A ∪ B, then x ∈ A ∪ B.  (←) Suppose x ∈ B, thus x ∈ A ∪ B.  That is, x ∈ A ⊆ B, x ∈ A ∪ B, and x ∈ B.  Therefore, if A ⊆ B, then A ∪ B = B. |

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| 2.5-5  “Show: A ∪ Ac = *U*” (Ferland, 2009, p. 98). Let A be an arbitrary set.  Consider A ∪ Ac = *U.*  (→) A ∪ Ac ⊆ U.  (←) Let x ∈ U.  Case 1, x ∈ A and x ∉ Ac.  Case 2, x ∉ A and x ∈ Ac.  In either case, x ∈ A ∪ Ac.  Therefore, A ∪ Ac = *U.* |
| 2.5-7  “Show: A ∪ U = U” (Ferland, 2009, p. 98).  Let A be an arbitrary set.  Consider A ∪ U = U  (→)A ∪ U ⊆ U.  (←) Let x ∈ U.  That is x ∈ A ∪ U.  Therefore A ∪ U = U. |

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| 2.5-9  “Show: A ∪ B ⊆ C if and only if A ⊆ C and B ⊆ C” (Ferland, 2009, p. 98).  Let A, B, and C be arbitrary sets.  Consider A ∪ B ⊆ C iff A ⊆ C and B ⊆ C.  (→) Let x ∈ A, or x ∈ B.  That is, x ∈ A ∪ B.  Since A ∪ B ⊆ C, then x ∈ C as well.  (←)  Case 1 Let x ∈ A.  Since A ⊆ C, then x ∈ C as well.  Case 2 Let x ∈ B.  Since B ⊆ C, then x ∈ C as well.  In all instances x ∈ C.  Therefore, A ∪ B ⊆ C iff A ⊆ C and B ⊆ C. |
| 2.5-13  “Show: A ∪ (B\C) = (A ∪ B)\ (C\A)” (Ferland, 2009, p. 98).  Let A, B, and C be arbitrary sets.  Consider A ∪ (B\C) = (A ∪ B)\ (C\A).  (→) Let x ∈ A.  Since A ∪ (B\C), specifically B\C, x ∈ A ∪ B.  (←) Let x ∈ A.  That is x ∈ A or x ∈ B.  Since C\A, x ∉ C, specifically x ∉ C\A.  Such that, x ∈ (A ∪ B)\ (C\A).  In either instance, x exists.  Therefore, A ∪ (B\C) = (A ∪ B)\ (C\A). |
| 2.5-15  “Show: ” (Ferland, 2009, p. 98).  Let A and B be arbitrary sets.  Consider.  Let (x, y) ∈ ↔ x ∈ Ac and y ∈ Bc ↔  (x, y) ∈ ∨ (x, y) ∈ .  That is, x ∈ and y ∈ .  In either instance, x ∈ and y ∈ (Note that (x, y) always exists in U).  Therefore, . |

Reference

Ferland, K. (2009). *Discrete Mathematics.* Boston: Houghton Mifflin Company.