Running Head: LOGIC AND SETS

Week 1 Application: Logic and Sets

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Week 1 Application: Logic and Sets

Exercises 1.1-8, 1.2-19, 1.3-4, 1.4-13, and 1.5-11 for grading:

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| 1.1-8  Create a truth table for the below statement  (p∨¬p)→r   |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | |  |  |  | 1 |  | answer |  | | **p** | **r** | **p** | **∨** | **¬p** | **→** | **r** | | f | f | f | t | t | f | f | | f | t | f | t | t | t | t | | t | f | t | t | f | f | f | | t | t | t | t | f | t | t | |
| 1.2-19  Express the given set in interval notation: “The set of real numbers x such that -1 < x and 1 > x” (Ferland, 2009, p. 31).  (-1,1) = {x: x ∈ R and -1 < x < 1} |
| 1.3-4  Write the following statement using quantifiers and standard notation: There is an integer n such that 2^n = 1024  ∃ n ∈ Z such that = 1024 |
| 1.4-13  Find {1,3}X{2,4}  (1,2),(1,4),(3,2),(3,4)} |
| 1.5-11  Determine whether the given argument form is valid or invalid:  p∨q  p→r  ∴¬p   |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | **p** | **q** | **r** | p ∨ q | p→r | ¬p | | f | f | f | f | t | t | | f | f | t | f | t | t | | f | t | f | t | t | t | | f | t | t | t | t | t | | t | f | f | t | f | f | | t | f | t | t | t | f | | t | t | f | t | f | f | | t | t | t | t | t | f |   Honestly, I am a bit confused by this one. The book says this form is valid, but the truth table says it isn’t. Have I misunderstood what the above says? The only way I can think of this being a valid form is if by ¬p being false in the last line, then it is what the statement says and therefore valid. However, this goes against what every other truth table says in the book. As such this statement is invalid as there is an instance when the premises are true, but the conclusion is false. |

Chapter 0, Odd numbered exercises 1 through 49:

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| 1.  To transform a base 2 to base 10 we must first multiply each digit by its corresponding 2^nth degree, where nth is the numerical position of the digit starting at 0 from the right. Once this is done, we add the values together. To confirm the value is correct, we divide it by 2, taking the reminder until the quotient is 0. Reverse the order of remainders and the value returned should be the original base 2.  Base 2: 1010 =  (1\*2^3)+(0\*2^2)+(1\*2^1)+(0\*2^0) =  8+0+2+0 =  **Base 10: 10** =  10%2= 0  5%2= 1  2%2= 0  1%2= 1  1010 |
| 3.  Base 2: 10111 =  (1\*2^4)+(0\*2^3)+(1\*2^2)+(1\*2^1)+(1\*2^0) =  16+0+4+2+1 =  **Base 10: 23 =**  23%2= 1  11%2= 1  5%2= 1  2%2= 0  1%2= 1  10111 |
| 5.  Base 2: 101110 =  (1\*2^5)+(0\*2^4)+(1\*2^3)+(1\*2^2)+(1\*2^1)+(0\*2^0)=  32+0+8+4+2+0=  **Base 10: 46 =**  46%2= 0  23%2= 1  11%2= 1  5%2= 1  2%2= 0  1%2= 1  101110 |
| 7.  Base 2: 1001011 =  (1\*2^6)+(0\*2^5)+(0\*2^4)+(1\*2^3)+(0\*2^2)+(1\*2^1)+(1\*2^0)=  64+0+0+8+0+2+1 =  **Base 10: 75 =**  75%2= 1  37%2= 1  18%2= 0  9%2= 1  4%2= 0  2%2= 0  1%2= 1  1001011 |

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| 9.  Base 2: 10101011 =  (1\*2^7)+(0\*2^6)+(1\*2^5)+(0\*2^4)+(1\*2^3)+(0\*2^2)+(1\*2^1)+(1\*2^0)=  128+0+32+0+8+0+2+1 =  **Base 10: 171 =**  171%2= 1  85%2= 1  42%2= 0  21%2= 1  10%2= 0  5%2= 1  2%2= 0  1%2= 1  10101011 |
| 11.  Base 10: 59 =  59%2= 1  29%2= 1  14%2= 0  7%2= 1  3%2= 1  1%2= 1  **Base 2: 111011 =**  (1\*2^5)+ (1\*2^4)+ (1\*2^3)+ (0\*2^2)+ (1\*2^1)+ (1\*2^0)=  32+16+8+0+2+1=  Base 10: 59 |
| 13.  Base 10: 84 =  84%2= 0  42%2= 0  21%2= 1  10%2= 0  5%2= 1  2%2= 0  1%2= 1  **Base 2: 1010100 =**  (1\*2^6)+ (0\*2^5)+ (1\*2^4)+ (0\*2^3)+ (1\*2^2)+ (0\*2^1)+ (0\*2^0)=  64+0+16+0+4+0+0=  Base10: 84 |
| 15.  Base 10: 117 =  117%2= 1  58%2= 0  29%2= 1  14%2= 0  7%2= 1  3%2= 1  1%2= 1  **Base 2: 1110101** =  (1\*2^6)+ (1\*2^5)+ (1\*2^4)+ (0\*2^3)+ (1\*2^2)+ (0\*2^1)+ (1\*2^0)=  64+32+16+0+4+0+1=  Base 10: 117 |

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| 17.  Base 10: 304 =  304%2= 0  152%2= 0  76%2= 0  38%2= 0  19%2= 1  9%2= 1  4%2= 0  2%2= 0  1%2= 1  **Base 2: 100110000 =**  (1\*2^8)+ (0\*2^7)+ (0\*2^6)+ (1\*2^5)+ (1\*2^4)+ (0\*2^3)+ (0\*2^2)+ (0\*2^1)+ (0\*2^0)=  256+0+0+32+16+0+0+0+0=  Base 10: 304 | |
| 19.  Base 10: 1024 =  1024%2= 0  512%2= 0  256%2= 0  128%2= 0  64%2= 0  32%2= 0  16%2= 0  8%2= 0  4%2= 0  2%2= 0  1%2= 1  **Base 2: 10000000000 =**  (1\*2^10)+ (0\*2^9)+ (0\*2^8)+ (0\*2^7)+ (0\*2^6)+ (0\*2^5)+ (0\*2^4)+ (0\*2^3)+ (0\*2^2)+ (0\*2^1)+ (0\*2^0)=  1024+0+0+0+0+0+0+0+0+0+0 =  Base 10: 1024 | |
| 21.  All of the possible outcomes from a sequence of 4 coin flips, e.g. 2^4  TTTT,TTTH,TTHT,TTHH,THTT,THTH,THHT,THHH,HTTT,HTTH,HTHT,HTHH,HHTT,HHTH,HHHT,  HHHH  T =Tails  H = Heads |
| 23.  Base 8 is slightly different to Base 2, in that, when converting from Base 10 to Base 8 the last value in the remainder sequence must be divided within an Integer function. This means, the value returned of a/b is of an integer value and not a real number. Also, when converting from Base 8 to base 10 we use the numerical value of 8 for the base value in (a\* b^n).  Base 8: 163 =  (1\*8^2)+(6\*8^1)+(3\*8^0)=  64+48+3=  **Base 10: 115 =**  115%8= 3  14%8= 6  INT(14/8)= 1  Base 8: 163 |

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| 25.  Base 8: 3217 =  (3\*8^3)+ (2\*8^2)+ (1\*8^1)+ (7\*8^0)=  1536+128+8+7=  **Base 10: 1679 =**  1679%8 = 7  209%8= 1  26%8= 2  INT(26/8)= 3  Base 8: 3217 |
| 27.  Base 8: 40510 =  (4\*8^4)+ (0\*8^3)+ (5\*8^2)+ (1\*8^1)+ (0\*8^0)=  16384+0+320+8+0=  **Base 10: 16712 =**  16712%8= 0  2089%8= 1  261%8= 5  32%8= 0  INT(32/8)= 4  Base 8: 40510 |
| 29.  Base 16 is similar to Base 8 in its remainder conversion. However, the values returned must be taken into the Hexadecimal context of:  a=10 b= 11 c=12 d=13 e=14 f=15  Base 16: dc9 =  (13\*16^2)+ (12\*16^1)+ (9\*16^0)=  3328+192+9=  **Base 10: 3529=**  3529%16= 9=9  220%16= 12=c  INT(220/16)= 13=d  Base 16: dc9 |
| 31.  Base 16: 5a7e =  (5\*16^3)+ (10\*16^2)+ (7\*16^1)+ (14\*16^0)=  20480+2560+112+14=  **Base 10: 23166 =**  23166%16= 14=e  1447%16= 7=7  90%16= 10=a  INT(90/16)= 5=5  Base 16: 5a7e |
| 33.  Base 10: 59 =  59%8= 3  INT(59/8)= 7  **Base 8: 73 =**  (7\*8^1) + (3\*8^0) =  56+3=  Base 10: 59 |

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| 35.  Base 10: 117 =  117%8= 5  14%8= 6  INT(14/8)= 1  **Base 8: 165 =**  (1\*8^2)+ (6\*8^1) + (5\*8^0) =  64+48+5=  Base 10: 117 |
| 37.  Base 10: 59 =  59%16= 11=b  INT(59/16)= 3=3  **Base 16: 3b =**  (3\*16^1)+(11\*16^0)=  48+11=  Base 10: 59 |
| 39.  Base 10:117 =  117%16= 5=5  INT(117/16)= 7=7  **Base 16: 75 =**  (7\*16^1)+(5\*16^0)=  112+5=  Base 10: 117 |
| 41.  Base 10: 44252 =  44252%16= 12=c  2765%16 = 13=d  172%16 = 12=c  INT(172/16) = 10=a  **Base 16: acdc =**  (10\*16^3)+ (12\*16^2)+ (13\*16^1)+ (12\*16^0)=  40960+3072+208+12=  Base 10: 44252 |
| 43. s=8 and s=16  Base 2: 1100110011   |  |  |  | | --- | --- | --- | | For s=8: Base 2: 001 100 110 011  001=  1  100=  (1\*2^2)+0+0= 4  110=  (1\*2^2)+ (1\*2^1)+0=4+2+0= 6  011=  0+(1\*2^1)+ (1\*2^0)=0+2+1= 3  **Base 8: 1463 =**  1 4 6 3  001 4%2=0 6%2=0 3%2=1  2%2=0 3%2=1 1%2=1  1%2=1 1%2=1 0  100 110 011 Base 2: 1100110011 | For s=16 Base 2: 0011 0011 0011  0011=  0+0+(1\*2^1)+(1\*2^0)=3  0011=  0+0+(1\*2^1)+(1\*2^0)=3  0011=  0+0+(1\*2^1)+(1\*2^0)=3  **Base 16: 333 =**  3 3 3  3%2=1 3%2=1 3%2=1  1%2=1 1%2=1 1%2=1  0 0 0  0 0 0  0011 0011 0011  001100110011  Base 2: 1100110011 | When converting from binary to octal, or hexadecimal, we split the binary sequence up into components matching the degree for which 2^n equals, starting from the right.  e.g. octal = 2^3  hexadecimal =2^4  If the binary sequence does not meet the length exactly, we pad the left most sequence with zeros. We then convert each separately, combining the numeric sequence at the end. | |
| 45.  Base 2: 101100110001011   |  |  | | --- | --- | | For s=8  Base 2: 101 100 110 001 011  101= (1\*2^2)+ (0\*2^1)+ (1\*2^0)=5  100=(1\*2^2)+ (0\*2^1)+ (0\*2^0)=4  110=(1\*2^2)+ (1\*2^1)+ (0\*2^0)=6  001=(0\*2^2)+ (0\*2^1)+ (1\*2^0)=1  011=(0\*2^2)+ (1\*2^1)+ (1\*2^0)=3  **Base 8: 54613 =**  5 4 6 1 3  5%2=1 4%2=0 6%2=0 1%2=1 3%2=1  2%2=0 2%2=0 3%2=1 0 1%2=1  1%2=1 1%2=1 1%2=1 0 0  101 100 110 001 011  Base 2: 101100110001011 | For s=16  Base2: 0101 1001 1000 1011  0101=(0\*2^3)+(1\*2^2)+(0\*2^1)+(1\*2^0)=5=5  1001=(1\*2^3)+(0\*2^2)+(0\*2^1)+(1\*2^0)=9=9  1000=(1\*2^3)+(0\*2^2)+(0\*2^1)+(0\*2^0)=8=8  1011=(1\*2^3)+(0\*2^2)+(1\*2^1)+(1\*2^0)=11=b  **Base 16= 598b =**  5 9 8 b  5%2=1 9%2=1 8%2=0 11%2=1  2%2=0 4%2=0 4%2=0 5%2=1  1%2=1 2%2=0 2%2=0 2%2=0  0 1%2=1 1%2=1 1%2 =1  0101 1001 1000 1011  Base 2: 101100110001011 | |
| 47.  Base 8: 47 =  4 7  4%2=0 7%2=1  2%2=0 3%2=1  1%2=1 1%2=1  100 111  **Base 2: 100111 =**  100=(1\*2^2)+(0\*2^1)+(0\*2^0)=4+0+0=4  111=(1\*2^2)+(1\*2^1)+(1\*2^0)=4+2+1=7  Base 8: 47 |
| 49: s=2  Base 16: acc =  a c c  10%2=0 12%2=0 12%2=0  5%2=1 6%2=0 6%2=0  2%2=0 3%2=1 3%2=1  1%2=1 1%2=1 1%2=1  1010 1100 1100  **Base 2: 101011001100** =  1010=(1\*2^3)+ (0\*2^2)+ (1\*2^1)+ (0\*2^0)=8+0+2+0=10=a  1100=(1\*2^3)+ (1\*2^2)+ (0\*2^1)+ (0\*2^0)=8+4+0+0=12=c  1100=(1\*2^3)+ (1\*2^2)+ (0\*2^1)+ (0\*2^0)=8+4+0+0=12=c  Base 16: acc |

Chapter 1.1 Odd numbered exercises 1-9

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| 1.1-1  Statement: “A sentence that is either true or false, but not both” (Ferland, 2009, p. 9).  “-1 is an integer” (Ferland, 2009, p. 21).  This is a statement as it has a definitive true or false value.  This statement is true. Integers are “the set consisting of the whole numbers, and their negative counterparts” (Blitzer R. , 2009). |
| 1.1-3  If π > 0, then compute  This is not a statement as it is asking for a computation and does not make a true or false claim. |
| 1.1-5  Create a truth table for the following statement: p ∨ ¬p   |  |  |  |  | | --- | --- | --- | --- | |  |  | Answer |  | | **p** | **p** | **∨** | **¬p** | | f | f | t | t | | f | f | t | t | | t | t | t | f | | t | t | t | f | |
| 1.1-7  Create a truth table for the following statement: ¬p → ( q ∧ r )   |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | |  |  |  |  | answer |  | 1 |  | | **p** | **q** | **r** | **¬p** | **→** | **q** | **∧** | **r** | | f | f | f | t | f | f | f | t | | f | f | t | t | f | f | f | f | | f | t | f | t | t | t | t | t | | f | t | t | t | f | t | f | f | | t | f | f | f | t | f | f | t | | t | f | t | f | t | f | f | f | | t | t | f | f | t | t | t | t | | t | t | t | f | t | t | f | f | |
| 1.1-9  Create a truth table for the following statement: (p→q)∨r   |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | |  |  |  |  | 1 |  | answer |  | | **p** | **q** | **r** | p | then | q | or | r | | f | f | f | f | t | f | t | f | | f | f | t | f | t | f | t | t | | f | t | f | f | t | t | t | f | | f | t | t | f | t | t | t | t | | t | f | f | t | f | f | f | f | | t | f | t | t | f | f | t | t | | t | t | f | t | t | t | t | f | | t | t | t | t | t | t | t | t | |

Section 1.2: Odd numbered exercises 1-27 (Note that 19 is listed at the top of this paper).

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| 1.2-1  Is the equality true or false?  {1,2,3}={3,2,1}  The equality is true as in set notation those items listed in a set are in no particular order.  All three numbers in the first set are listed in the second set. |
| 1.2-3  {2,2,2,2}={2,2}  The equality is true as all numbers in the first set exist in the second set. |
| 1.2-5  Express the set in set notation  2,4,6  {2,4,6} |
| 1.2-7  The set of {1},{4}  {{1},{4}} |
| 1.2-9  The set of: (x^3)+(4x^2)+(5x)-(6)=0  **{x:x∈R and (x^3)+(4x^2)+(5x)-(6)=0}**  (x^3)+(4x^2)+(5x)=6  3 NAN  **{3}** |
| 1.2-11  The set of integers less than -10  {p:p∈Z and p < -10} |
| 1.2-13  Express in interval notation: R+  (0,∞) |
| 1.2-15  Express in interval notation: {0}  [0,0] starts at 0 and ends at 0 |
| 1.2-17  Express in interval notation: {x:x∈R and 1<x}  (1,∞) |
| 1.2-21  True or False:  True, the square root of 2 is a real number. |
| 1.2-23  True or false: {1}∈ Z  False, while 1 is an integer by itself, the set of 1 is not explicitly an integer. |
| 1.2-25  True or false: {2}⊆{1,2,3}  True, the set of 2 is a subset of the set 1,2,3. |
| 1.2-27  True or false: θ∈{θ}  False, the empty set, θ, is not an element of the set of θ. |

Section 1.3, odd exercises 1-11,21,23, and 25

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| 1.3-1  Write the following statement using quantifiers and standard notation:  “There is an integer whose reciprocal is also an integer” (Ferland, 2009, p. 39).  ∃ n ∈ Z such that 1/n ∈ Z |
| 1.3-3  “For every real number x, is positive” (Ferland, 2009, p. 39).  ∀x∈R, |
| 1.3-5  “There is a natural number n such that, for every real number x, is nonnegative” (Ferland, 2009, p. 39).  ∃ n ∈ N such that ∀ x ∈ R, |
| 1.3-7  “There is a real number x such that, for every real number y with 2≤y≤3, we have 1 ≤xy<2” (Ferland, 2009, p. 40).  ∃x∈R such that ∀y∈R, (2≤y≤3) → (1 ≤xy<2) |
| 1.3-9  “There are real numbers z and y such that x+y ∈ Z and xy ∉ Z” (Ferland, 2009, p. 40).  ∃ x, y ∈ R such that x+y ∈ Z and xy ∉ Z |
| 1.3-11  “Express the fact that the exponential function f(x) = is increasing in a precise statement using quantifiers” (Ferland, 2009, p. 40).  ∀ x, y ∈ R, (x < y) → () |
| 1.3-21  Negate the given statement:  There is an integer whose reciprocal is also an integer  ¬[∃ n ∈ Z such that 1/n ∈ Z] ≡∀ n ∈ Z, 1/n ∉ Z |
| 1.3-23  Negate the given statement:  For every real number x, is positive  ¬[∀ x ∈ R, ]≡∃ x ∈ R such that |
| 1.3-25  Negate the given statement:  There is a natural number n such that, for every real number x, is nonnegative  ¬[∃ n ∈ N such that ∀ X ∈ R, > 0] ≡ ∀ n ∈ N, ∃ x ∈ R such that |

Section 1.4, Odd exercises 1-19 (Note that question 13 is located at the top of this paper).

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| 1.4-1  Find  Where: A={1,2,3}, B = {3,4}, and U = {1,2,3,4} |
| 1.4-3  Find  Where: A=(-1,1), B=[0,1], and U = R |
| 1.4-5  Find  Where: A=N, B=, and U = Z |
| 1.4-7  Find  Where: A=(0,3), B=[2,∞), and U = |
| 1.4-9  Are Z- and N disjoint  Yes, Z- encompasses all negative integers, while N encompasses all positive natural numbers. |
| 1.4-11  Is [2,4] a disjoint union  No, [2,4] |
| 1.4-15  Find {3,5,7,9}X{5}  {(3,5),(5,5),(7,5),(9,5)} |
| 1.4-17  Sketch the subset of R^2 for [2,4]X[1,3]  {(2,1),(2,3),(4,1),(4,3)}   |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | | 6 |  |  |  |  |  |  | | 5 |  |  |  |  |  |  | | 4 |  |  |  |  |  |  | | 3 |  |  |  |  |  |  | | 2 |  |  |  |  |  |  | | 1 | 1 | 2 | 3 | 4 | 5 | 6 | | -1 |  |  |  |  |  |  | |

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| 1.4-19  Sketch the given subsets of R^2 for: (-1,1)^2  {(-1,-1),(-1,1),(1,-1),(1,1)}   |  |  |  |  | | --- | --- | --- | --- | |  | 2 |  |  | |  | 1 | 1 | 2 | | -2 | -1 |  |  | |  | -2 |  |  | |

Section 1.5 Odd Exercises 9-13 (note that question 11 appears at the top of this paper).

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| 1.5-9  Determine whether the given argument form is valid or invalid:  p→q  q  ∴ p   |  |  |  |  |  | | --- | --- | --- | --- | --- | | p | q | p → q | q | p | | f | f | t | f | f | | f | t | t | t | f | | t | f | f | f | t | | t | t | t | t | t |   Invalid, there is an instance when both premises are true but the conclusion is false. |
| 1.5-13  Determine whether the given argument form is valid or invalid:  p∨q  p  ∴¬q   |  |  |  |  |  | | --- | --- | --- | --- | --- | | p | q | p or q | p | not q | | f | f | f | f | t | | f | t | t | f | f | | t | f | t | t | t | | t | t | t | t | f |   Invalid, there is an instance when both premises are true but the conclusion is false. |

Reference

Blitzer, R. (2009). *Introductory and Intermediate Algebra for College Students* (3rd ed.). Upper Saddle River, NJ, United States of America: Pearson Prentice Hall.

Blitzer, R. (2009). *Introductory and Intermediate Algebra for College Students (3rd ed.).* Upper Saddle River, NJ, United States of America: Pearson Prentice Hall.

Ferland, K. (2009). *Discrete Mathematics.* Boston: Houghton Mifflin Company.